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| Ratios, Rates, and Percents |
| Mathematics Grade 6 |
|  |
| The focus of this unit is to develop an understanding of ratios and rates. Students learn that ratios compare the same types of measures and represent part:whole and part:part relationships. They also learn that ratios that compare different types of measures are called rates. Students apply these concepts to a variety of real world and mathematical situations, including problems involving measurement conversions and percents. In the culminating performance task, students plan a recipe, using ratios to find the quantities, unit rates, and costs of ingredients for different numbers of servings.  This unit addresses the grade 6 Critical Area #1 of the 2011MA Curriculum Framework for Mathematics: *Connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems.*  *These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.* |

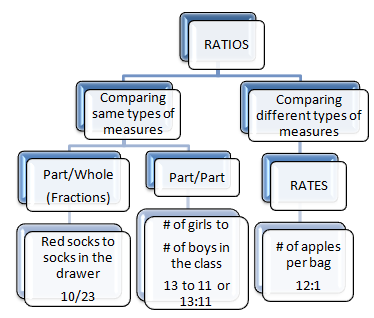


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| Table of Contents  [Stage 1 Desired Results 3](#_Toc365024025)  [Stage 2 - Evidence 7](#_Toc365024026)  [Stage 3 – Learning Plan 8](#_Toc365024027)  [Lesson 1: Introduction to Ratios 10](#_Toc365024028)  [Lesson 2: Writing Ratios 15](#_Toc365024029)  [Lesson 3: Equivalent Ratios 26](#_Toc365024030)  [Lesson 4: Equivalent Ratios 37](#_Toc365024031)  [Lesson 5 Solving Mathematical and Real-Life Problems with Ratios 50](#_Toc365024032)  [Lesson 6: Ratios - Assessment 62](#_Toc365024034)  [Lesson 7: Understanding Rates and Unit Rates 74](#_Toc365024035)  [Lesson 8: Solving Problems with Unit Rates 81](#_Toc365024036)  [Lesson 9 Using Rates and Unit Rates to Make Informed Consumer Deciisons 86](#_Toc365024037)  [Lesson 10: Rates and Unit Rates Assessment 94](#_Toc365024039)  [Lesson 11: What is a Percent? 99](#_Toc365024040)  [Lesson 12: Percents and Tape Diagrams 116](#_Toc365024041)  [Lesson 13: Solving Percent Problems: Missing Part, Missing Percent 122](#_Toc365024042)  [Lesson 14: Solving Percent Problems: Missing Whole 126](#_Toc365024043)  [Curriculum Embedded Performance Assessment (CEPA) 141](#_Toc365024044)  [List of Unit Resources 151](#_Toc365024046) |

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| Stage 1 Desired Results | | | | | |
| ESTABLISHED GOALS  **6.RP.1**  Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*  **6.RP.2**  Understand the concept of a unit rate ***a***/***b*** associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”* [[1]](#footnote-1)  **6.RP.3**  Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.  Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.  Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*  Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means **30**/**100** times the quantity); solve problems involving finding the whole, given a part and the percent.  Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.  **Standards for Mathematical Practice:**  **G4 SMP.1**  **Make sense of problems and persevere in solving them.**  (Students must understand the problem  context in order to translate them into ratios/rates.)  **G5 SMP.2**  **Reason abstractly and quantitatively**.  (Students must understand the relationship  between two quantities in order to express them mathematically.)  **G6 SMP.3**  **Construct viable arguments and critique the reasoning of others.**  This supports **ELA 6.W.1: Write arguments to support claims with clear reasons and relevant evidence.**  (This will be reinforced in class discussion and in the written requirements of the CEPA.)  **G7 SMP.4**  **Model with mathematics.**  (Students can model a real-life situation using  ratios and rates.)  **G8 SMP.7**  **Look for and make use of structure.**  (The structure of a ratio is unique and can be used across a wide variety of problem-solving situations.)  *Supporting Standards*:  **RST.4**  Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6 – 8 texts and topics.  **WHST.2**  Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.   1. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. 2. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. 3. Use precise language and domain-specific vocabulary to inform about or explain the topic. 4. Establish and maintain a formal style and objective tone. 5. Provide a concluding statement or section that follows from and supports the information or explanation presented.   **WHST.4**  Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. | | | | ***Transfer*** | |
| *Students will be able to independently use their learning to…*  Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems. | |
| ***Meaning*** | |
| UNDERSTANDINGS  *Students will understand that…*  **U1**  A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities (including unit rates.)  **U2**  A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.  (See the chart on p.6)  **U3**  Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.) | ESSENTIAL QUESTIONS  **Q1**  When is it useful to be able to relate one quantity to another?  **Q2**  How are ratios and rates used in everyday life? How would life be different without ratios and rates?  *Note:*  *Essential questions provide an important anchor for the unit. They should be posted in the classroom throughout the unit. Students can be redirected to these questions throughout the unit lessons through questions, discussions, and math journal entries.* |
| ***Acquisition*** | |
| *Students will know…* **K**  **K1**  A **ratio** compares two related quantities.  **K2**  Ratios can be represented in a variety of formats including *each, to, per, for each, %, 1/5*, :, etc.  **K3**  A **percent** is a type of ratio that compares a quantity to 100.  **K4**  A **unit** **rate** is the ratio of two measurements in which the second term is 1.  **K5**  When it is appropriate to use ratios/rates to solve mathematical or real life problems.  **K6**  Mathematical strategies for solving problems involving ratios and rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc. | *Students will be able to…* **S**  **S1**  Use ratio and rate reasoning to solve real-world and mathematical problems.  **S2**  Make and interpret tables of equivalent ratios.  **S3**  Plot pairs of values of the quantities being compared on the coordinate plane.  **S4**  Use multiple representations such as tape diagrams, double number line diagrams, or equations to solve rate and ratio problems.    **S5**  Solve unit rate problems (including unit pricing and constant speed).  **S6**  Solve percent problems, including finding a percent of a quantity as a rate per 100 and finding the whole, given the part and the percent. |
| Stage 2 - Evidence | | | | | |
| **Evaluative Criteria** | | | **Assessment Evidence** | | |
| See CEPA rubric. | | | **Curriculum Embedded Performance Assessment**  **Title**: Pizza Champions  **Overview**  For this assessment, students assume the role of school cafeteria chef to create a pizza recipe and plan ingredients to make pizza for 12, 60, and 240 students. Students compute unit prices for the ingredients, prepare a budget, and calculate the cost to feed varying numbers of people. Using ratio/rate language, each student writes a proposal to persuade the cafeteria manager to use the recipe.  **List of Large-Scale Tasks**  CEPA Task I: Cost to Make One Pizza  CEPA Task II: Cost to Feed 12, 60, and 240 Students  CEPA Task III: Proposal to the Cafeteria Manager  All materials for this CEPA can be found at the end of the unit. | | |
| Other  Assessments: | **Lesson 2**  **Lesson 3**  **Lesson 6**  **Lesson 7**  **Lesson 9**  **Lesson 10**  **Lesson 14** | Ticket-to-Leave: *Are All Fractions Ratios?*  Quiz: *Equivalent Ratios*  Summative Assessment: *Ratios*  Formative Assessment: *Rates*  Formative Assessment: *Unit Rates: Justin’s Pizza*  Summative Assessment: *Unit Prices: Little Red Riding Hood’s Grocery Trip*  Summative Assessment: *Percent Problems: Tim’s Tires, Raffle Ticket Sales, and Library Books* | | | |
| Stage 3 – Learning Plan | | | | | |
| *Summary of Key Learning Events and Instruction*  Note: In Grade 6, the focus is on ratios, equivalent ratios, and rates. In Grade 7, students will study proportions and proportional reasoning. All sessions are 50 minutes each.   |  |  |  | | --- | --- | --- | | Lesson 1 | Introduction to Ratios | 1 session | | Lesson 2 | Writing Ratios | 1 session | | Lesson 3 | Equivalent Ratios Part 1 | 1 session | | Lesson 4 | Equivalent Ratios Part 2 | 1 session | | Lesson 5 | Solving Mathematical and Real-Life Problems with Ratios | 2 sessions | | Lesson 6 | Ratios: Review and Assessment | 2 sessions | | Lesson 7 | Understanding Rates and Unit Rates | 1 session | | Lesson 8 | Solving Problems with Unit Rates | 1 session | | Lesson 9 | Using Rates / Unit Rates to Make Good Consumer Decisions | 1 session | | Lesson 10 | Rates and Unit Rates Assessment | 1 session | | Lesson 11 | What is a Percent? | 1 session | | Lesson 12 | Percents and Tape Diagrams | 1 session | | Lesson 13 | Solving Percent Problems: Missing Part, Missing Percent | 1 session | | Lesson 14 | Solving Percent Problems: Missing Whole | 2 sessions | | Lesson 15 | Curriculum-Embedded Performance Task (CEPA) | 3-5 sessions | | | | | | |

Types of Ratios

(See UNDERSTANDINGS , p.2)



## Lesson 1: Introduction to Ratios

**Brief Overview of Lesson:**

Students learn that a ratio compares two quantities. They use visual models and real-world examples to gain concrete understanding of how ratios are used. There is a brief introduction to key vocabulary and notation. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students will be able to multiply fractions.
* Students are able to find equivalent fractions without manipulatives.
* Academic vocabulary: compare, quantity.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Websites

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Grade 8 Mathematics**

**Lesson # and title:** Introduction to Ratios

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson, students will know:*

* A ratio compares two related quantities.
* Ratios can be represented in a variety of formats, including *1 to 5, 1:5, 1/5*.

*By the end of this lesson, students will be able to:*

* Use a ratio to express the relationship between two quantities.

**Essential Question(s) addressed in this lesson:**

* When is it useful to be able to relate one quantity to another?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, *“The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly 3 votes.”*

**6.SMP.2**Reason abstractly and quantitatively. (Students must understand the relationship between two quantities in order to express them in ratio form.)

**Teacher Content and Pedagogy**

**Ratio:**

A ratio expresses the relationship between two quantities. Ratios compare two measures of the same types of things. Examples: *the number of one color of marble to another color of marbles, or the number of cats to dogs.*

Ratios can compare parts to a whole (part:whole). Example: *12 of the 15 students are playing soccer (12/15).*

Ratios can also compare a part of one whole to another part of the same whole (part:part). Example: *The ratio of green marbles in the jar to red marbles in the jar is 4:2.*

Ratios can be expressed in following notation: *x:y, x/y, or x to y.*

***Note: Rates and unit rates are addressed later in this unit.***

**Rate:**

When a ratio compares two different types of measures, it is called a rate. Examples: *5 gallons of paint are needed to paint 8 walls (5:8). 3 shirts for $20 (3/$20)*

**Unit Rate:**

A unit rate is a rate which compares a quantity to one of the other quantity. Examples: *Miles per hour, cost per foot, eggs per carton.*

**Proportion:**

(not addressed until Gr. 7)

A proportion is an equation written in a form that states that two ratios are equal. *A/B = C/D*

**Anticipated Student Preconceptions/Misconceptions:**

* Students may be confused about the order of the quantities. For example, a comparison of 2 wins to 3 losses is written as 2:3, and not 3:2. It is helpful if students begin labeling the quantities of the things they are comparing both in writing and orally.
* Students may have difficulty distinguishing a part:part ratio from a part:whole ratio. For example, *“There are 12 girls compared to 11 boys in the class (12:11), but 12 of the 23 students in the class are girls (12:23).”*

**Lesson Sequence:**

1. Introduce students to the Math Wall for the unit. (see explanation at end of lesson)

Ask probing questions like the ones given below either through a math journal entry or small group discussion to help students reason abstractly and quantitatively (SMP.2); i.e. students should explicitly understand the relationship between two quantities in order to express them in the ratio form (questions c and d).

* 1. What is a ratio?
  2. Why do we use ratios?
  3. What are some examples of real-world ratios?
  4. What do those ratios mean?

1. Collect student ideas about ratios and post on chart paper or the board without comment or corrections.
2. Explain that a ratio compares two quantities. Ask students to work in pairs to find some ratios in the classroom (without requiring specific ratio language or notation). Give an example or two to get them started. Students write down their ideas, and then share out. Examples: *chairs to desks, students to teachers, books per students, the number of boys to girls, etc.*
3. Ask students what the ratios mean. For example if the ratio of books to students is 1:2, what does this mean? (It might be difficult for two students to share a book.) What are the implications of either a very high teacher:student ratio or a very low teacher:student ratio?
4. Briefly introduce the concept of ratio and the key vocabulary and notation associated with it.
5. View a teaching video on ratios.

At the learnalberta.ca website there are a variety of teaching videos. This is a Mathematics-Grade 6 Spy Guys Ratio video.

[http://www.learnalberta.ca/content/mesg/html/math6web/index.html?](http://www.learnalberta.ca/content/mesg/html/math6web/index.html?page=lessons&lesson=m6lessonsshell03.swf)page=lessons&lesson=m6lessonsshell03.swfThis interactive mathematics lesson teaches students about ratios. Students apply ratios in real life situations, such as making recipes larger and calculating distance using the scale on a map.

and/or <http://www.youtube.com/watch?v=eT1yYqmjHPY>

or <http://www.youtube.com/watch?v=yztq_ELjfSw&feature=related>

(Teaching videos focusing on the definition of ratio and the ways that a ratio can be expressed.)

1. Revisit student ideas:
   1. Are there any ideas that need to be refined based on the activities and the videos?
   2. Do students want to contribute additional thoughts to the chart about ratios?
   3. Do they want to delete flawed ideas from the chart?

9. Extended Learning/Practice (homework)

Students find three examples of ratios in the real world. They can find examples on the internet, in newspapers, or in their own homes. For each, they write down the ratio and discuss its meaning. Example: *The ratio of citizens who voted in the last election compared to those who didn’t vote was 1:6. Analysis: Not very many people voted. A few people are making decisions for the whole city.* Example: *Two of my sisters have jobs after school. The ratio of their hourly pay is $7:$10. Analysis: The sister who makes $7 an hour could ask for a raise in her hourly rate, but she is younger and has less experience, so it is probably fair.*

**Formative assessment: None**

**Summative assessment: None**

**Preview outcomes for the next lesson:**

* Students will be able to write ratios to express the relationship between two quantities.
* Students will use appropriate ratio notation and language.

**Math Walls**

At the beginning of this unit, select a wall or bulletin board as the designated space for the Math Wall and display the title, “Ratio and Rates.” The Math Wall will serve as an interactive resource to support students as they develop their mathematical understanding of ratio reasoning.

As the unit unfolds and each lesson is taught, add the following content:

* Vocabulary words, as they are introduced in the day’s lesson.
* Pictures, diagrams, manipulatives …multiple ways to represent the mathematical thinking.
* Strategies, problem solving steps, ways to communicate our thinking, ways to represent a number.
* Sample student work, examples of journal entries.
* Criteria/rubrics

A Math Wall….

* Assists students in understanding the mathematical language they may come across in written and spoken instructions.
* Serves as a valuable resource for students to refer to.
* Demonstrates how mathematics builds on what is already known.
* Provides a [context](http://www.teacher-support-force.com/learning-context.html) for the learning taking place in the classroom.
* Makes the mathematics more **accessible** by allowing students to recall the appropriate vocabulary, work expectations, accurate spelling, representations, and strategies.
* Provides the support for concepts and processes based upon students’ needs.

## Lesson 2: Writing Ratios

**Brief Overview of Lesson:**

Students practice writing part : part and part : whole ratios in three formats: 1to 4, 1:4, and ¼. They pay attention to the order of the quantities. For example, the ratio of 1 cup of sugar to 4 cups of flour should be written as 1:4, and not 4:1. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Multiplying fractions
* Finding equivalent fractions.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Pictures of objects in scattered arrangements (provided by teacher)
* Websites
* Lesson 2: *In-Class Work*
* Lesson 2: *Homework*

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 2:** Writing Ratios

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* A ratio expresses the relationship between two related quantities.
* Ratios can be represented in a variety of formats including *1 to 5, 1/5, 1:5*..

*By the end of this lesson, students will be able to:*

* Write a ratio to express the relationship between two quantities.
* Use proper ratio notation and language to describe the relationship between two quantities.

**Essential Question(s) addressed in this lesson:**

* When is it useful to be able to relate one quantity to another?
* How are ratios used in everyday life?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, *“The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly 3 votes.”*

**6.SMP.4** Model with mathematics. Students model real-life situations with mathematics.

**Teacher Notes**

In this lesson, the focus is on writing ratios that accurately represent mathematical, tabular, or pictorial situations. In the next lesson, students will be asked to express ratios in simplest form.

Here are some additional technology resources for all students. These resources facilitate students model with mathematics (SMP.4) The first one may be very useful for students with disabilities or ELLs.

1. [http://www.thinkingblocks.com/ThinkingBlocks\_Ratios/ TB\_Ratio\_Main.html](http://www.thinkingblocks.com/ThinkingBlocks_Ratios/%20TB_Ratio_Main.html)

Interactive site where students are taught how to use blocks to model ratio problems. Problems may ask students to find one of the two quantities in the ratio, the difference between the two quantities, or the total. Provides a video with step-by-step clear, visual, auditory demonstration of using blocks to solve ratio problems. Teachers can use to guide instruction with block manipulatives , or students can virtually manipulate blocks.

1. [http://illuminations.nctm.org/LessonDetail.aspx? id=L722](http://illuminations.nctm.org/LessonDetail.aspx?%20id=L722)

Pairs (or groups) of students use a cup of beans to find ratios to express the number of marked beans in the cup compared to the total number of beans in the cup. Theoretically, each sample should be essentially the same. The decimal representation of each ratio confirms that ratios are, indeed, approximately equivalent.

3. <http://www.math-aids.com.Ratios/>

Practice sheets that use shapes to help students explore ratio relationships.

Note: Some students may benefit from using actual manipulatives that they can move around.

**Anticipated Student Preconceptions/Misconceptions: ---**

**Lesson Sequence:**

1. Students work in groups of 3 to briefly share their “ratios in the real world homework”. Each group shares one good example with the rest of the class. Student 1 reads the example so that the teacher can record it on the board. Student 2 tells which notation to use in the written ratio. Student 3 explains the meaning of the ratio and any inferences that can be made. This activity reviews the previous lesson, and pre-assesses student readiness for writing ratios.
2. Students practice writing ratios based on the following types of problems:

* Pictures of objects in scattered arrangements.

Questions should include **part: whole**:

Write the ratio that compares the number of \_\_\_\_ to the total number of \_\_\_\_\_\_ in the \_\_\_\_\_.

Questions should also include **part:part**:

Write the ratio that compares the number of \_\_\_\_\_ to the number of \_\_\_\_\_\_.

Questions should request ratios in **different notation** (e.g., write as x:y, write as x/y, express as x to y, etc.)

* + Working backward: In the diagram, what does the ratio \_\_\_:\_\_\_ represent
* Shapes divided into equal parts with some parts shaded. Write the ratios for shaded to non-shaded or shaded to total.

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1. Ticket to Leave:

We know that all ratios can be written in fraction form.

*Are all fractions ratios? Why or why not?*

(Students write a response, explaining their thinking on a card or paper scrap. After putting their names on them, they turn them in on their way out.)

1. Extended Learning/Practice (homework)

Students write ratios in the form requested for 6 different situations.

Students create a ratio problem for someone else to solve. Include pictures of objects, the question (what ratio the student wants the solver to find), and the form in which the student wants the ratio written.

**Formative assessment:**

Ticket to Leave (see above)

**Summative assessment: None**

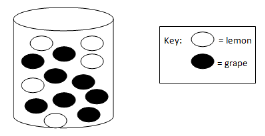
**Preview outcomes for the next lesson:**

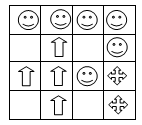
Students will learn to make tables of equivalent ratios.

Students will identify missing values in a table of equivalent ratios.

**Lesson 2: In-Class Work**

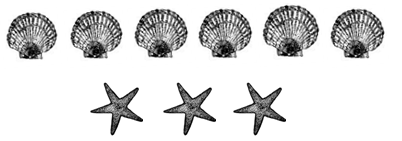
1. The jar below contains grape and lemon jelly beans.



1. What is the ratio of lemon jelly beans to grape jelly beans?
2. How many grape jelly beans would have to be added so that the ratio was 1 lemon jelly bean for 2 grape jelly beans?
3. Show this solution by drawing the added grape jelly beans in the jar. How many jelly beans are in the jar now?
4. With the added jelly beans, write the ratio of lemon jelly beans to the whole jar of jelly beans in fraction form.
5. With the added jelly beans, write the ratio of grape jelly beans to the whole jar of jelly beans in fraction form.
6.  Here is a grid with different kinds of boxes.
7. Using the colon form (:), write the ratio of to \_\_\_\_\_\_\_\_
8. Using the fraction form, write the ratio of to the total number of boxes in the grid. \_\_\_\_\_
9. Write the ratio of to the total number of boxes in the grid. \_\_\_\_\_\_\_\_
10. Write the ratio of empty boxes to the total number of boxes in the grid. \_\_\_\_\_\_
11. Write the ratio of to \_\_\_\_\_\_\_
12. Which two kinds of boxes have a ratio of 2:3? Justify your answer.
13. Luisa has 12 playing cards. She places 2 out of every 3 cards face-up on the table.

= face-up = face-down

1. Draw Luisa’s cards, showing how many cards are face-up and how many are face-down.
2. Luisa wants to change the ratio to 2 face-up cards to 3 face-down cards. How many cards does she need to add to the original 12 cards? Draw the solution.
3. Dr. Jin has 6 scallops and 3 sea stars in his aquarium.

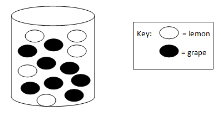


1. What is the ratio of sea stars to scallops? \_\_\_\_\_\_\_\_\_\_\_\_\_
2. Each sea star eats 4 scallops per week. How many scallops does Dr. Jin need to add to his tank to feed the sea stars for a week? \_\_\_\_\_\_\_\_\_\_\_\_\_
3. One of Dr. Jin’s fellow scientists brings him two more sea stars. What will the ratio of sea stars to scallops need to be in order to feed the sea stars for a week? \_\_\_\_\_\_\_\_\_\_\_\_\_

**Lesson 2: In-Class Work ANSWER KEY**

(Ratios in fraction form do not need to be in lowest terms. In this lesson, it is more important for students to write a comparison of two quantities correctly.)

1. The jar below contains grape and lemon jelly beans.



1. What is the ratio of lemon jelly beans to grape jelly beans? **5:8, 5/8, 5 to 8**
2. How many grape jelly beans would have to be added so that the ratio was 1 lemon jelly bean for 2 grape jelly beans? **Add 2 more grape jelly beans: 5/10 = ½ (Accept 5/10)**
3. Show this solution by drawing the added grape jelly beans in the jar. How many jelly beans are in the jar now?

**15 jelly beans in all**

1. With the added jelly beans, write the ratio of lemon jelly beans to the whole jar of jelly beans in fraction form.

**5/15 = 1/3 (Accept 5/15)**

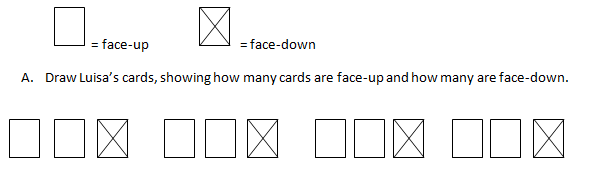
1. With the added jelly beans, write the ratio of grape jelly beans to the whole jar of jelly beans in fraction form.

**a four by four grid with six smiley faces, four arrows pointing up, two crosses, and four blank squares
10/15 = 2/3**

1. Here is a grid with different kinds of boxes.
2. Using the colon form (:), write the ratio of to **2:4 or 1:2**
3. Using the fraction form, write the ratio of to the total number of boxes in the grid. **6/16 or 3/8**
4. Write the ratio of to the total number of boxes in the grid. **4/16 or 1/4**
5. Write the ratio of empty boxes to the total number of boxes in the grid. **4/16 or 1/4**
6. Write the ratio of to **2/6 or 1/3**
7. Which two kinds of boxes have a ratio of 2:3? Justify your answer. **Arrow to smiley face**

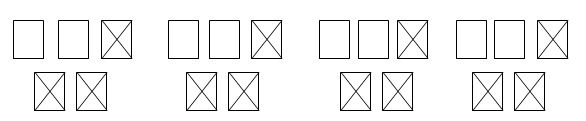
**Lesson 2: In-Class Work ANSWER KEY**

1. Luisa has 12 playing cards. She places 2 out of every 3 cards face-up on the table.

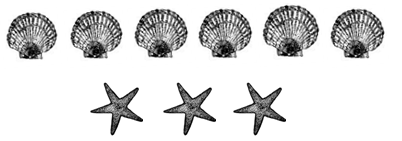


B. Luisa wants to change the ratio to 2 face-up cards to 3 face-down cards. How many cards does she need to add

to the original 12 cards? Draw the solution.

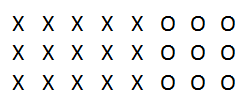


1. Dr. Jin has 6 scallops and 3 sea stars in his aquarium.



1. What is the ratio of sea stars to scallops? **3 to 6 3:6 3/6 or 1/2**
2. Each sea star eats 4 scallops per week. How many scallops does Dr. Jin need to add to his tank to feed the sea stars for a week?  **6 more scallops**
3. One of Dr. Jin’s fellow scientists brings him two more sea stars. What will the ratio of sea stars to scallops need to be in order to feed the sea stars for a week? **5 sea stars : 20 scallops**

**Lesson 2: Homework**





What is the ratio of X to O in this set? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

In one row, what is the ratio of X to O? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What is the ratio of O to the whole set? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

In one row, what is the ratio of O to X? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Here is a set of cards:

a set of five playing cards face down.

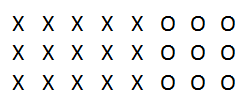
Write each ratio as a fraction:

* 1. The ratio of face-up cards to the whole set.
  2. The ratio of face-down cards to the whole set.
  3. The ratio of face-down cards to face-up cards.
  4. Add a drawing of some face-down cards so that the ratio of face-down to face-up cards is 3:4. Justify your answer.

1. Samantha drew a 1:4 ratio. Stephan drew a ratio picture that had more than nine objects, but it was still a 1:4 ratio. Draw a picture to show what Stephan could have drawn.
2. Create a ratio problem for someone else to solve. Include pictures of objects.

Your problem should clearly tell the problem-solver what ratio to find, and in what form the answer should be written (to, fraction, colon).

**Lesson 2: Homework ANSWER KEY**





What is the ratio of X to O in this set? **15 to 9 15:9 15/9**

In one row, what is the ratio of X to O? **5 to 3 5:3 5/3**

What is the ratio of O to the whole set? **9 to 24 9:24 9/24**

In one row, what is the ratio of O to X? **3 to 5 3:5 3/5**

1. Here is a set of 13 cards:

a set of five playing cards face down

Write each ratio as a fraction:

* 1. The ratio of face-up cards to the whole set. **8 to 13 8:13 8/13**
  2. The ratio of face-down cards to the whole set. **5 to 13 5:13 5/3**
  3. The ratio of face-down cards to face-up cards. **5 to 8 5:8 5/8**
  4. Add a drawing of some face-down cards so that the ratio of face-down to face-up cards is 3:4. Justify your answer. **One more face-down card is needed. There will be 6 face down cards to 8 face up cards.**

1. Samantha drew a 1:4 ratio. Stephan drew a ratio picture that had more than nine objects, but it was still a 1:4 ratio. Draw a picture to show what Stephan could have drawn.

**Accept any that shows a 1:4 ratio**.

1. Create a ratio problem for someone else to solve. Include pictures of objects.

Your problem should clearly tell the problem-solver what ratio to find, and in what form the answer should be written (to, fraction, colon). **Accept any form.**

## Lesson 3: Equivalent Ratios

**Brief Overview of Lesson:**

Students make the connection between equivalent fractions and equivalent ratios. They find missing values in a table of equivalent ratios, establishing the multiplicative relationship first rather than looking for iterative patterns in the table. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

1. Students can create and complete tables.
2. Students can multiply multi-digit numbers.
3. Students understand that equivalent fractions are different names for the same quantity.
4. Students can use different strategies to find equivalent fractions.
5. Students understand that a fraction a/b is a multiple of 1/b.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

Equivalent ratios quiz printed out from <http://www.math-aids.com/Ratios> (See description below in Formative Assessment)

Lesson 3 *Homework*

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics, Grade 8**

**Lesson 3: Equivalent Ratios**

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:* Mathematical strategies for solving problems involving ratios, including tables, and equivalent fractions.

*By the end of this lesson, students will be able to:*

* Make tables of equivalent ratios.
* Find missing values in tables of equivalent ratios.

**Essential Question(s) addressed in this lesson:**

* Are all fractions ratios? Why or why not?
* How can I use what I know about fractions to solve ratio problems?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, and find missing values in the table.

Use tables to compare ratios.

**6.MP.7** Look for and make use of structure.

**Assumptions about what students know and are able to do coming into this lesson (including language needs):**

* Students can create and complete tables.
* Students can multiply multi-digit numbers.
* Students understand that equivalent fractions are different names for the same quantity.
* Students can use different strategies to find equivalent fractions.
* Students understand that a fraction a/b is a multiple of 1/b.

**Teacher Notes**

A ratio expresses the relationship between two quantities.

It is important for middle school students to understand that the relationship is a **multiplicative relationship**.

A **multiplicative relationship** is defined as a relationship in which one number is multiplied to get the second number. In addition, the same multiplicative relationship can be applied to other situations. It yields a relative rather than an absolute number.

*For example, if the ratio of boys to girls in a school is 2:3, the school could have 20 boys and 30 girls, or 200 boys and 300 girls, or 400 boys and 600 girls. Given any number of boys, you can find the number of girls by multiplying the boys by 3/2. Given any number of girls, you can find the number of boys by multiplying by 2/3. In essence, these ratios can all be expressed as equivalent fractions. Note: Through working problems in this lesson student will be required to look closely and discern patterns and structures. In this lesson students will*

Students learn fractions by viewing shapes that are divided into equal parts with some of the parts shaded. They have an **additive understanding** of fractions.

For example, in a circle divided into fourths with three parts shaded, students think:

¾ = ¼ + ¼ + ¼

They see the 3 parts shaded and they see the 4 parts total in the circle, and they think “3 out of 4 parts are shaded”. However, this makes it difficult for students to think about quantities such as 5/3. It also makes it difficult for them to apply knowledge of fractions to proportional reasoning.

In middle school, students should view ¾ as a multiplicative relationship: 3 x ¼ . They need to understand a/b as a x 1/b parts.  
This multiplicative understanding of fractions begins in grade 4 (4.NF.4a).

If students understand that ratios have a multiplicative relationship, they will be able to use tables, equivalent fractions, and tape diagrams to solve ratio problems.

When finding missing values in a table, students should be encouraged to establish the ratio first, rather than looking for iterative patterns in the table.

To check student understanding of a multiplicative relationship, give students the following problem.

Julie swims 1 lap for every 3 laps that Joe swims. Eliza made a table showing this relationship:

Julie

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 4 | 7 | 10 |
| 3 | 6 | 9 | 12 |

Joe

Is Eliza correct? (She is not – she has used a reiterative addition pattern of +3 rather than a multiplicative relationship to solve the problem.)

The correct table is: Julie

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 4 | 7 | 10 |
| 3 | 12 | 21 | 30 |

Joe

*In working on this and other problems in this lesson students will be looking for and making use of structure (SMP.7).*

Important note: Teachers should avoid teaching the “cross multiply and divide” strategy of solving proportions in Gr. 6.

**Anticipated Student Preconceptions/Misconceptions:**

* Understanding the multiplicative relationship of ratios can be difficult for many students. Teachers should spend significant time with hands-on activities and guided practice, while modeling multiple strategies to help students see this relationship. (See Teacher Content Knowledge section.)
* In the context of a ratio problem, a ratio is not necessarily a fraction. For example, ¾ relates a quantity of 3 to a quantity of 4. If this is a part-to-whole comparison, then the ratio is a fraction. If it is a part-to-part comparison, it is not a fraction.

**Lesson Sequence:**

1. Class discussion based on the Ticket to Leave from the last lesson:

*Ratios can be written as fractions. Are all fractions ratios?* (Can students think of counterexamples? Students present arguments based on mathematical thinking as they debate this.)

Note: All fractions are indeed ratios which express multiplicative relationships. *For example, 1/2 = 2/4 = 3/6, etc. In this example, you can multiply any numerator x 2 to get the denominator.*

1. Relate the idea of equivalent ratios to equivalent fractions. If 540/630 students like pepperoni on their pizza, what does this mean? To make the relationship easier to understand, find an equivalent ratio by expressing 540/630 in simplest terms: 6/7 students like pepperoni pizza.
2. Game: Students find ratios between two quantities of objects.   
   <http://math.rice.edu/~lanius/proportions/rate5.html>  
   For example, students choose which picture shows 2 dolls for every 1 camera (4:2)
3. Show a table of ratio values, with a value missing. Here is an example:

|  |  |
| --- | --- |
| Cost of Potatoes | |
| Pounds of Potatoes | Cost |
| 2 | $3.00 |
| 4 | $6.00 |
| 6 | ? |
| 8 | ? |

To find the missing value, it is important to establish the ratio first. In this case, the ratio is 2:3. Students discuss how to find the missing value.

For students who need more scaffolding, present a table with a simpler ratio, and no “skipped” quantities in the left-hand column, such as the following:

|  |  |
| --- | --- |
| Cost of Grapes | |
| Pounds of Grapes | Cost |
| 1 | $2.00 |
| 2 | $4.00 |
| 3 | $6.00 |
| 4 | ? |
| 5 | $10.00 |

For students who need additional challenge, present a table with no labels and missing values. Ask students to create a context for the table presented, add labels, and find the missing values.

|  |  |
| --- | --- |
| Title: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | |
|  |  |
| 2 | 30 |
| 5 | ? |
| ? | 120 |
| 10 | 150 |

1. Students work with a partner to create a similar table problem.
2. Quiz with equivalent ratio problems in this format:

2 : 3 = 4 : \_\_ = 6 : \_\_ = 8 : \_\_ = 10: \_\_ = 12: \_\_   
 (Can be downloaded and printed out from: <http://www.math-aids.com/Ratios> )

1. Extended Learning/Practice (homework)

Students fill in missing values in a ratio table and make a table of equivalent ratios.

**Formative assessment:** Ratio Quiz (see #7)

**Preview outcomes for the next lesson:** Students will learn to plot the values of quantities being compared on the coordinate plane.

**Lesson 3: Homework**

1. Michaela charges $6.50 per hour to babysit. Complete the table below to show how much she would earn.

|  |  |
| --- | --- |
| **Babysitting Earnings** | |
| **Hours Worked** | **Amount Earned** |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

1. At birth, a baby blue whale weighs 5,000 pounds. The baby whale continues to gain 200 pounds per day. Make a table showing how much the baby whale gains in one week. Remember to include a title and label each column.

|  |  |
| --- | --- |
|  | |
|  |  |
| Day 0 (birth) | 5000 |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

1. What will the baby whale weigh at the end of 10 days? How did you find your answer?
2. Explain how to determine how much the baby whale will weigh on any day. How could you find the weight on the 20th day and the 50th day?
3. The drama club mixes paint needed for scenery. To make green, the ratio is 2 pints of blue per 3 pints of yellow. Fill in the missing numbers below. Explain your thinking.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Mixing Green Paint** | | | | | | | |
| BLUE | 1 | 2 |  |  | 20 |  | 50 |
| YELLOW |  | 3 | 9 | 15 |  | 60 |  |

1. It takes Hans 6 hours to read 480 pages. Today, he is going to read for 5 hours. How many pages will he read? Fill out the table below to figure out how many pages Hans will read.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Pages Read** | | | | |
| HOURS | 1 | 2 |  | 6 |
| PAGES |  |  | 240 | 480 |

1. For every 3 laps that Julie ran, Joe ran 9 laps.
   1. When Julie finished 1 lap, how many laps had Joe run? \_\_\_\_\_\_\_\_\_\_\_\_\_
   2. When Julie finished 9 laps, how many laps had Joe run? \_\_\_\_\_\_\_\_\_\_\_\_
   3. When the total number of laps that Julie and Joe have completed is 16, how many of those laps were Joe’s, and how many were Julie’s? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
   4. Make a table below to show how you got your answers. Remember to include a title and labels for each column.

**Lesson 3: Homework ANSWER KEY**

1. Michaela charges $6.50 per hour to babysit. Complete the table below to show how much she would earn.

|  |  |
| --- | --- |
| **Babysitting Earnings** | |
| **Hours Worked** | **Amount Earned** |
| 1 | **$6.50** |
| 2 | **$13.00** |
| 3 | **$19.50** |
| 4 | **$26.00** |
| 5 | **$32.50** |
| 6 | **$39.00** |

2. At birth, a baby blue whale weighs 5,000 pounds. The baby whale continues to gain 200 pounds per day. Make a table showing how much the baby whale gains in one week. Remember to include a title and label each column.

|  |  |
| --- | --- |
| **Weight Gain of a Baby Whale** | |
| **Days** | **Pounds** |
| **Day 0 (birth)** | **5000** |
| **Day 1** | **5200** |
| **Day 2** | **5400** |
| **Day 3** | **5600** |
| **Day 4** | **5800** |
| **Day 5** | **6000** |
| **Day 6** | **6200** |
| **Day 7** | **6400** |

3. What will the baby whale weigh at the end of 10 days? How did you find your answer?

**7000 lbs. (Extend the table or multiple 200 by the number of days and add to 5000)**

4. Explain how to determine how much the baby whale will weigh on any day. How could you find the weight on the 20th day and the 50th day?

**20th day:** **5000 + (200 x 20) = 9000 50th day:** **5000 + (200 x 50) = 15,000**

5. The drama club mixes paint needed for scenery. To make green, the ratio is 2 pints of blue per 3 pints of yellow. Fill in the missing numbers below. Explain your thinking.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Mixing Green Paint** | | | | | | | |
| BLUE | 1 | 2 | **6** | **10** | 20 | **40** | 50 |
| YELLOW | **1.5** | 3 | 9 | 15 | **30** | 60 | **75** |

6. It takes Hans 6 hours to read 480 pages. Today, he is going to read for 5 hours. How many pages will he read? Fill out the table below to figure out how many pages Hans will read.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Pages Read** | | | | |
| HOURS | 1 | 2 | **3** | 6 |
| PAGES | **80** | **160** | 240 | 480 |

7. For every 3 laps that Julie ran, Joe ran 9 laps.

1. When Julie finished 1 lap, how many laps had Joe run? **3 laps**
2. When Julie finished 9 laps, how many laps had Joe run? **3/9 = 9/27 27 laps**
3. When the total number of laps that Julie and Joe have completed is 16, how many of those laps were Joe’s, and how many were Julie’s? **Julie ran 4 laps and Joe ran 12 laps**
4. Make a table below to show how you got your answers. Remember to include a title and labels for each column. **(See sample table)**

**JULIE AND JOHN’S LAPS**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Julie’s Laps** | 1 | 2 | 3 | 4 |
| **Joe’s Laps** | 3 | 6 | 9 | 12 |
| **Total Laps** | 4 | 8 | 12 | 16 |

## Lesson 4: Equivalent Ratios

**Brief Overview of Lesson:**

Students plot values of equivalent ratios on the coordinate plane, and use these graphs to solve problems. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students should understand that equivalent fractions are different names for the same quantity.
* They should be able to use different strategies to find equivalent fractions.
* They should be able to plot a point (*x,y*) on a coordinate plane.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Lesson 4: *In-Class Work*
* Lesson 4: *Homework*
* Websites
* Graph paper or coordinate plane paper

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Grade 8 Mathematics**

**Lesson 4: Equivalent Ratios**

**Time (minutes):** 50

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:* Mathematical strategies for solving problems involving ratios, including tables, equivalent fractions, and graphs.

*By the end of this lesson, students will be able to:* Plot pairs of values of the quantities being compared on the coordinate plane.

**Essential Question(s) addressed in this lesson:** How can the relationship between two quantities be represented visually?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

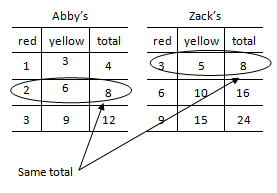
**6.MP.7** Look for and make use of structure.

**Teacher Notes**

When equivalent ratios are graphed, the data points always form a straight line. The equation of the line is in the form *y = mx + b*, where *m* is the slope or the multiplicative factor. It can also be thought of as the rate of change. This will be taught in grade 8 (8.F.4). (*b* is the y-intercept, indicating where the line crosses the y axis when x = 0)

In advance, make graphs that can be shown on an overhead projector, document camera, etc. This will allow students to quickly check the accuracy of their work. There are many technology tools that can provide quick graphs. If students do not have correct values in their graphs, the lesson will be ineffective.

Remind students that information can be gained from comparing the different values in the tables. For example, here are three ways to look at a table comparing paint ratios.

**Six tables showing ways Abby and Zack mixed paint mixtures.In the first two tables,  arrowd are drawn to show that Abby and Zack had the same amount of red, but the proportions were different due to the differing amounts of yellow.
In the second set of tables, arrows point out the same amount of yellow used by Abby and Zack. The third set of tables  haved arrows denoting the same total parts, but diffferent amounts of yellow and red paint.**

Note: When students explore the above problem they are actually engaging in SMP.7 (Look for and make use of structure.).

**Anticipated Student Preconceptions/Misconceptions:**

Translating a ratio into an ordered pair to be plotted on the coordinate plane may be difficult for many students. Some students lack basic graphing skills, such as distinguishing the x axis from the y axis, or plotting points with the coordinates in the correct order.

In addition, they may view graphing as entirely unrelated to ratios. Teachers need to make the connection that graphing provides another way of looking at the relationship between ratio values.

**Lesson Sequence:**

Check students’ graphing skills by doing a quick graphing activity. The teacher gives a series of coordinates in quadrant one: (4,1), (2,3), (4,5) and (6,3).

If students plot the points correctly, it will form a square.

1. A. Students find at least 3 equivalent ratios for 8/12, and make a simple ratio table. On an overhead projector, graph students’ numbers, using the first quantity (the numerator) as the x coordinate, and the second quantity (the denominator) as the y coordinate. In the above example, students might volunteer these coordinates: (16, 24), (2, 3), and (4, 6). What do students notice about the resulting line? (The graph is a straight line, indicating a proportional relationship.)

B. Students read the graph to find two additional points on the line. They identify the coordinates of the two points. Can students justify that these points represent equivalent ratios? Extension for advanced students: Can students make a generalization about all of the points on the line?

1. Use the graph from Part B to evaluate whether their answers to Problem 5 from the previous night’s homework are correct.
2. Revisit the completed data tables from the previous lesson. Be sure that data tables have the correct values, as noted in bold below.

|  |  |
| --- | --- |
| Cost of Potatoes | |
| Pounds of Potatoes | Cost |
| 2 | $3.00 |
| 4 | $6.00 |
| 6 | **$9.00** |
| 8 | **$12.00** |

|  |  |
| --- | --- |
| Cost of Grapes | |
| Pounds of Grapes | Cost |
| 1 | $2.00 |
| 2 | $4.00 |
| 3 | $6.00 |
| 4 | **$8.00** |
| 5 | $10.00 |

|  |  |
| --- | --- |
| Title: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | |
|  |  |
| 2 | 30 |
| 5 | **75** |
| **8** | 120 |
| 10 | 150 |

1. Students graph the values in these data tables (or any other examples). What do they notice? (Again, the resulting graph is a straight line.)
2. The teacher shows how to use the graph to find a missing value. For example, in a graph based on the first data table, students could find an interim data point to estimate how much 5 lbs.of potatoes cost. Or, they could extend the straight line to find out how much 20 lbs. of potatoes cost.

9. Students complete the In-Class Worksheet (2 problems).

**Pre-Assessment:**

Pre-Assessment of Graphing Skills (See beginning of Lesson Sequence)

**Preview outcomes for the next lesson:** Students will learn other strategies for solving mathematical and real-life problems with ratios.

**Lesson 4: In-Class Work**

1. James can read 12 pages in 4 minutes.
   1. Complete the following table:

|  |  |
| --- | --- |
| **Time to Read** | |
| **Time in Minutes** | **Pages Read** |
| 4 | 12 |
|  | 24 |
|  | 36 |
|  | 48 |
|  | 60 |

* 1. Use graph paper to plot the values from the data table. Draw a line that best fits the data points, extending it beyond the first and last data points.
  2. Use the graph to find the number of pages that can be read in 1 minute. Circle this point on the graph.
  3. Use the graph to find the number of pages that can be read in 20 minutes. Put a around this point.
  4. Does the graph show a ratio relationship between the pages read and the time in minutes? How do you know? Support your answer with evidence from the table and the graph.

1. Mrs. Baker poured a cup of hot chocolate and set it on the table to cool. She recorded the temperature of the hot chocolate every 10 minutes. Here is her data:

|  |  |
| --- | --- |
| **Hot Chocolate Temperature** | |
| **Elapsed Time in Minutes** | **Temperature** |
| 5 | 180 |
| 10 | 165 |
| 15 | 140 |
| 20 | 130 |
| 25 | 125 |
| 30 | 120 |
| 35 | 115 |
| 40 | 112 |

1. Use graph paper to plot the values from the chart.
2. Does the graph show a ratio? How do you know? Support your answer from the table and the graph.

**Lesson 4: In-Class Work ANSWER KEY**

1. James can read 12 pages in 4 minutes.
   1. Complete the following table:

|  |  |
| --- | --- |
| **Time to Read** | |
| **Time in Minutes** | **Pages Read** |
| 4 | 12 |
| **8** | 24 |
| **12** | 36 |
| **16** | 48 |
| **20** | 60 |

* 1. Use graph paper to plot the values from the data table. Draw a line that best fits the data points, extending it beyond the first and last data points.
  2. Use the graph to find the number of pages that can be read in 1 minute. Circle this point on the graph. **3 pages**
  3. Use the graph to find the number of pages that can be read in 20 minutes. Put a around this point. **60 pages**
  4. Does the graph show a ratio relationship between the pages read and the time in minutes? How do you know? Support your answer with evidence from the table and the graph.

**Yes, the graph shows a ratio relationship. Each point on the graph represents the ratio between two quantities. All points on the graph are equivalent ratios = 3/1. That is why they form a straight line. There is a constant change from point to point.**

1. Mrs. Baker poured a cup of hot chocolate and set it on the table to cool. She recorded the temperature of the hot chocolate every 10 minutes. Here is her data:

|  |  |
| --- | --- |
| **Hot Chocolate Temperature** | |
| **Elapsed Time in Minutes** | **Temperature** |
| 5 | 180 |
| 10 | 165 |
| 15 | 140 |
| 20 | 130 |
| 25 | 125 |
| 30 | 120 |
| 35 | 115 |
| 40 | 112 |

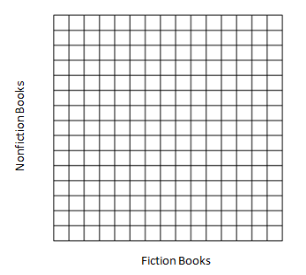
1. Use graph paper to plot the values from the chart.
2. Does the graph show a ratio relationship between elapsed time and temperature? How do you know? Support your answer from the table and the graph.

**No, this graph does not show a constant change from one point to the next. From 5 minutes to 10 minutes, the temperature dropped 15 degrees, but from 10 minutes to 15 minutes, the temperature dropped 25 degrees. These ratios are not equal. The points on the graph do not form a straight line.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Temperature** | 200 |  |  | |  | |  | |  | |  | |  | |  | |
| 180 |  |  | |  | |  | |  | |  | |  | |  | |
| 160 |  |  | |  | |  | |  | |  | |  | |  | |
| 140 |  |  | |  | |  | |  | |  | |  | |  | |
| 120 |  |  | |  | |  | |  | |  | |  | |  | |
| 110 |  |  | |  | |  | |  | |  | |  | |  | |
| 100 |  |  | |  | |  | |  | |  | |  | |  | |
| 80 |  |  | |  | |  | |  | |  | |  | |  | |
| 60 |  |  | |  | |  | |  | |  | |  | |  | |
| 40 |  |  | |  | |  | |  | |  | |  | |  | |
|  | 20 |  |  | |  | |  | |  | |  | |  | |  | |
|  | 0 | 5 | | 10 | | 15 | | 20 | | 25 | | 30 | | 35 | | 40 | |
|  | **Elapsed Time in Minutes** | | | | | | | | | | | | | | | | |

**Lesson 4: Hot Chocolate Temperature Graph**

**Lesson 4: Homework**

1. Complete the table and graph for the following situation: The library has 5 fiction books for every 3non-fiction books.

|  |  |
| --- | --- |
| **Books in the Library** | |
| **Fiction** | **Nonfiction** |
| 5 | 3 |
|  |  |
|  |  |

1. In April 2012, the U.S. dollar could buy 1.30 Euros. Make a table showing what $1, $5, $10, $15, $20, $50, and $100 would purchase in Euros. Graph this relationship. (Note: A “Euro” is a unit of money used in many European countries.)
2. Complete the following data table. Then make a graph that shows this relationship. How far can a car travel on 1 gallon of gasoline? Circle this point on your graph.

|  |  |
| --- | --- |
| **Gas Mileage** | |
| Miles | Gallons of Gas |
|  | 1 |
| 50 | 2 |
| 75 |  |
| 100 | 6 |

1. The recipe for Pumpkin Dog Biscuits is 2 eggs, ½ cup canned pumpkin, ½ teaspoon salt, 2 tablespoons dry milk, and 2 ½ cups wheat flour. This recipe makes about 20 treats. Rewrite the recipe for 10 biscuits. 50 biscuits. 100 biscuits. Show your work. What method did you use to rewrite the recipe? (Equivalent fractions, tape diagram, double line diagram?) Why did you choose that method?

**Lesson 4: Homework ANSWER KEY**

1. Complete the table and graph for the following situation:

The library has 5 fiction books for every 3 non-fiction books.

|  |  |
| --- | --- |
| **Books in the Library** | |
| **Fiction** | **Nonfiction** |
| **5** | **3** |
| **10** | **6** |
| **15** | **9** |

Graph showing the number of nonfiction books in relation to the fiction books. The vertical axis is numbered 0,2,4,6,8,10 and the horizontal axis is numbered 0,10,20


1. In April 2012, the U.S. dollar could buy 1.30 Euros. Make a table showing what $1, $5, $10, $15, $20, $50, and $100 would purchase in Euros. Graph this relationship. (Note: A “Euro” is a unit of money used in many European countries.)

|  |  |
| --- | --- |
| **US Dollars** | **Euros** |
| **1** | **1.30** |
| **5** | **6.50** |
| **10** | **13.00** |
| **15** | **19.50** |
| **20** | **26.00** |
| **50** | **65.00** |
| **100** | **130.00** |

1. Complete the following data table. Then make a graph that shows this relationship. How far can a car travel on 1 gallon of gasoline? Circle this point on your graph.

|  |  |
| --- | --- |
| **Gas Mileage** | |
| Miles | Gallons of Gas |
| **25** | 1 |
| 50 | 2 |
| 75 | **3** |
| 100 | 6 |

1. The recipe for Pumpkin Dog Biscuits is 2 eggs, ½ cup canned pumpkin, ½ teaspoon salt, 2 tablespoons dry milk, and 2 ½ cups wheat flour. This recipe makes about 20 treats. Rewrite the recipe for 10 biscuits. 50 biscuits. 100 biscuits. Show your work. What method did you use to rewrite the recipe? (Equivalent fractions, tape diagram, double line diagram?) Why did you choose that method?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **# of BISCUITS** | **EGGS** | **CANNED PUMPKIN**  **(cups)** | **SALT**  **(teaspoons)** | **DRY MILK**  **(tablespoons)** | **WHEAT FLOUR**  **(cups)** |
| 10 dog biscuits | 2 | 1/2 | 1/2 | 2 | 2 1/2 |
| 20 dog biscuits | 4 | 1 | 1 | 4 | 5 |
| 50 biscuits | 10 | 2 1/2 | 2 1/2 | 10 | 17 1/2 |
| 100 biscuits | 20 | 5 | 5 | 20 | 25 |

## Lesson 5

## Solving Mathematical and Real-Life Problems with Ratios

**Brief Overview of Lesson:**

Students use multiple representations such as tables of equivalent fractions, graphs, tape diagrams, and double line diagrams to solve ratio problems, including measurement conversion problems. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Assumptions about what students know and are able to do coming into this lesson (including language needs):
* Students should know basic unit measurement conversions for length (e.g., 12 inches in a foot; 3 feet in a yard; 100 cm in a meter), mass (e.g. 1000 grams in a kilogram), and volume (e.g., 1000 ml in a liter). For students who are not secure in this knowledge, it is helpful to provide a chart of conversions.

**Estimated Time (minutes):** Two 50 minute lessons

**Resources for Lesson:**

* Lesson 5: In-Class Practice
* Top-Down Web

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 5: Solving Mathematical and Real-Life Problems with Ratios**

**Time (minutes):** Two 50 minute lessons

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* When it is appropriate to use ratios to solve mathematical or real-world problems.
* Mathematical strategies for solving problems involving ratios, including tape diagrams and double line diagrams.

*By the end of this lesson students will be able to:*

* Use ratio reasoning to solve real-world and mathematical problems.
* Use multiple representations such as tape diagrams and double line diagrams to solve ratio problems.
* Convert measurement units using ratio reasoning

**Essential Question(s) addressed in this lesson:**

* How are ratios used in everyday life?
* What strategies can we use to solve ratio problems?
* What is the best way to communicate solutions of ratio problems?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate

plane. Use tables to compare ratios.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

**6.SMP.4** Model with mathematics. (Students model a real-life situation using ratios.)

**Teacher Notes**

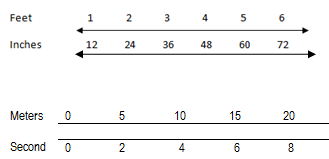
There is lots of new terminology in this lesson. Make sure that students understand and can apply the vocabulary.

It is important to recognize ratios in many different contexts. To beginners, each proportional reasoning situation presents a different kind of problem, even though they may understand the concept of ratios. More experienced students recognize that although ratios occur in many different settings, the same mathematical idea (multiplicative relationship) underlies all. These students have a toolbox of strategies for solving ratio problems.

**Double line diagrams:**

A double line diagram compares two quantities that have a multiplicative ratio.

Example:



**Tape diagrams:**

A tape diagram is a drawing that looks like a segment of tape, used to illustrate number relationships. They are also known as strip diagrams, bar models or graphs, fraction strips or length models.

The following examples exhibit modeling with mathematics (SMP.4). For example in problem one students use the tape measure as a tool to show the relationship between passenger cars and trucks and then apply this relationship mathematically to find the solution to the real life problem.

Example:

*A sixth grade class counted the number of vehicles that went by the front entrance of the school between 9 o’clock and 10 o’clock. The total number of car counted was 156. There were 3 times as many passenger cars as trucks. How many passenger cars and how many trucks were counted?*

Using this tape diagram, students can see that the ratio is 1:3. 156 needs to be divided into 4 equal parts. There were 39 trucks and 117 cars.

[](http://4.bp.blogspot.com/_MR07cM9jv68/S7U_K94RyFI/AAAAAAAAAUs/U9f1Y38XSLs/s1600/P)

A fruit punch recipe has a 3 to 2 ratio of apple juice to grape juice.

|  |  |  |  |
| --- | --- | --- | --- |
| Apple Juice |  |  |  |
| Grape Juice |  |  |  |

This diagram represents any mixture of apple juice and grape juice with a ratio of 3 to 2. The total amount of juice is partitioned into 5 parts of equal size, represented by the 5 shaded boxes.

For example, if the diagram represents 5 cups of juice, then each box represents 1 cup.

If the diagram represents 15 cups of juice, then each box represents 3 cups.

If the total amount of juice is 1 gallon, than each part represents 1/5 gallon and there are 3/5 gallon of apple juice and 2/5 gallon of grape juice.

**Extensions: The Golden Ratio**

Extensions: Students may be interested in ratios such as the golden ratio (found in art and nature), common ratios in architecture, and ratios in cooking and nutrition.

[http://www.geom.uiuc.edu/~demo5337/ s97b/art.htm](http://www.geom.uiuc.edu/~demo5337/%20s97b/art.htm)

Information on the GOLDEN RATIO. Includes pictures of the Parthenon, Pyramids, and other art works. Gives directions to draw plane figures based on the Golden Ratio.

<http://www.youtube.com/watch?v=fmaVqkR0ZXg&feature=youtube_gdata_player>

Overview of the Golden Ratio. Includes examples from math, art, architecture, and real-life objects. ~3 minutes

<http://www.nytimes.com/2009/04/29/dining/reviews/29book.html> An article about the book “Ratio: The Simple Codes Behind the Craft of Everyday Cooking” explains how ratios can be used to simplify cooking. Example: 5:3 ratio of flour to water in bread making.

**Anticipated Student Preconceptions/Misconceptions:**

Many students have not had enough experience with different types of measurement of length, mass, and volume. The U.S. system is difficult to understand and must be memorized. The metric system makes more sense, but it is easy to confuse with the U.S. system, which is more familiar to most students in terms of daily life experiences.

**Lesson Sequence:**

1. Ratios can be used to convert measurements. With students, the teacher makes a list on the board of common measurement ratios. Examples: 12 inches/foot, 3 feet/yard, 100 cm to a meter, 1000 grams/kilogram, etc.
2. The teacher demonstrates the use of **Double Line Diagrams** for converting measurements.
3. Students are given tables that list various measurements with missing values. Remind students about multiplicative relationships. Ask them to demonstrate that their table entries are correct by creating a double line diagram.

Example:

|  |  |
| --- | --- |
| **Inches** | **Feet** |
| 48 |  |
|  | 12 |
| 144 |  |

|  |  |
| --- | --- |
| **Grams** | **Kilograms** |
| 100 |  |
|  | 6 |
| 1300 |  |

1. The teacher demonstrates the use of **Tape Diagrams** to solve ratio problems. Through guided practice, students apply this strategy to several problems.
2. Students solve a variety of ratio problems, using tables, graphs, equivalent fractions, tape diagrams, and double line diagrams as directed.
3. Given a tape diagram, students create a story problem that matches the diagram, solve it themselves, and then try it out on a friend. (Teachers may need to provide feedback about the wording and the structure of the student-authored problem to ensure clarity before the problems are distributed to others.)
4. Students fill out the top-down web about ratios (see Formative Assessments). The teacher circulates to correct misconceptions.
5. Students are given a sample of student work in which the student has made an error in solving a ratio problem. Ask students to explain the error and correct it.
6. Extension: Students view examples of ratios in nature, art, etc. (See websites.)
7. Extended Learning/Practice (homework)

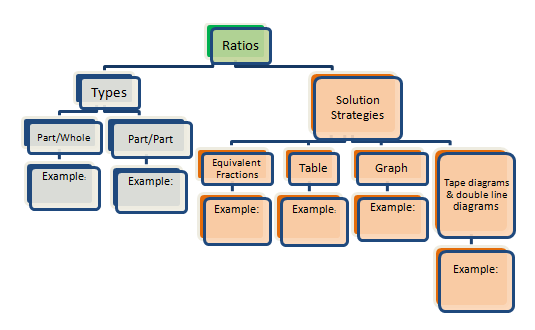
Journal entry:

When you are solving a ratio problem, which strategy makes the most sense to you?

* Equivalent fractions
* Graph
* Table
* Double Line Diagram
* Tape Diagram
* Other

Explain why this strategy is the best one for you.

**Formative assessment:**

See # 6: Given a template with the first three levels provided, students create a top-down web about ratios by providing examples. Students should be encouraged to go back and look at prior work. They work individually at first. Then they work with a partner to complete their webs.

**An example of a top-down web filled out: 
The first two links are types and solution strategies. TYpes then break down to part/whole and part/part with blanks for examples. Solution strategies break down to equivalent fractions, table, graph, tape diagrams and double line diagrams with examples. **

**Preview outcomes for the next lesson:**

* Students will apply knowledge of ratios to propose, justify, and communicate a solution (both orally and in writing).
* Students will prepare for a summative assessment on ratios.

**Lesson 5: In-Class Practice**

Use equivalent fractions, tape diagrams, or double line diagrams to solve the following problems.

1. Fill in the following table:

|  |  |
| --- | --- |
| **Linear Measurement Conversion** | |
| **Inches** | **Feet** |
| 48 |  |
|  | 5 |
| 132 |  |
|  | 12 |

1. Use the following double line diagram to represent the information in the table above.

Inches:

Feet:

1. The Green Team Field Trip  
   1. Mrs. Timmon’s Green Team Club has 39 students. They are taking a field trip to a park to study pond life. If the students travel by car, each car will carry 4 students. Make a table that shows the ratio of cars to students. How many cars will she need?
   2. Mrs. Timmons can also arrange to rent minivans. If the students travel by minivan, each minivan will carry 6 students. Make a table that shows the ratio of minivans to students. How many minivans will she need?
   3. Mrs. Timmons decides to rent a combination of cars and minivans. What is the most efficient combination of cars and minivans? Justify your answer.
2. Mixing Paints  
   1. To mix purple paint, an artist uses 2 quarts of red paint to 3 quarts of blue paint. Make a tape diagram to show how the artist mixes purple paint. Include labels.
   2. The artist mixes up 15 quarts of purple paint. How many quarts are red? How many parts are blue? Show your answer with a tape diagram.
   3. Check your answer with another method, and explain how this method justifies your original answer.
3. A runner and a bike rider leave Calder School at the same time. For every mile that the runner completes, the bike rider will complete 3 miles. If the runner completes 3 miles, how far will the bike rider travel? Show how you got your answer using two different methods.

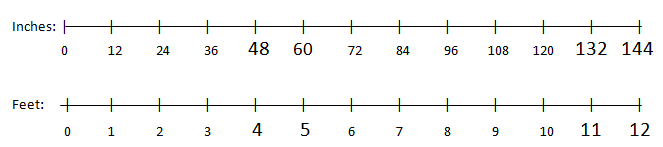
**Lesson 5: In-Class Practice ANSWER KEY**

Use equivalent fractions, tape diagrams, or double line diagrams to solve the following problems.

1. Fill in the following table:

|  |  |
| --- | --- |
| **Linear Measurement Conversion** | |
| **Inches** | **Feet** |
| **48** | **4** |
| **60** | **5** |
| **132** | **11** |
| **144** | **12** |

1. Use the following double line diagram to represent the information in the table above.



1. The Green Team Field Trip  
   1. Mrs. Timmon’s Green Team Club has 39 students. They are taking a field trip to a park to study pond life. If the students travel by car, each car will carry 4 students. Make a table that shows the ratio of cars to students. How many cars will she need? **10 cars**
   2. Mrs. Timmons can also arrange to rent minivans. If the students travel by minivan, each minivan will carry 6 students. Make a table that shows the ratio of minivans to students. How many minivans will she need? **7 minivans**
   3. Mrs. Timmons decides to rent a combination of cars and minivans. What is the most efficient combination of cars and minivans? Justify your answer.

**6 vans and 1 car 6 vans = 36 seats, and 1 car = 4 seats, for a total of 40 (39 students and 1 teacher.)**

1. Mixing Paints  
   1. To mix purple paint, an artist uses 2 quarts of red paint to 3 quarts of blue paint. Make a tape diagram to show how the artist mixes purple paint. Include labels.

|  |  |  |  |
| --- | --- | --- | --- |
| RED |  |  |  |
| BLUE |  |  |  |

* 1. The artist mixes up 15 quarts of purple paint. How many quarts are red? How many parts are blue? Show your answer with a tape diagram.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| RED |  |  |  |  |  |  |  |  |  |
| BLUE |  |  |  |  |  |  |  |  |  |

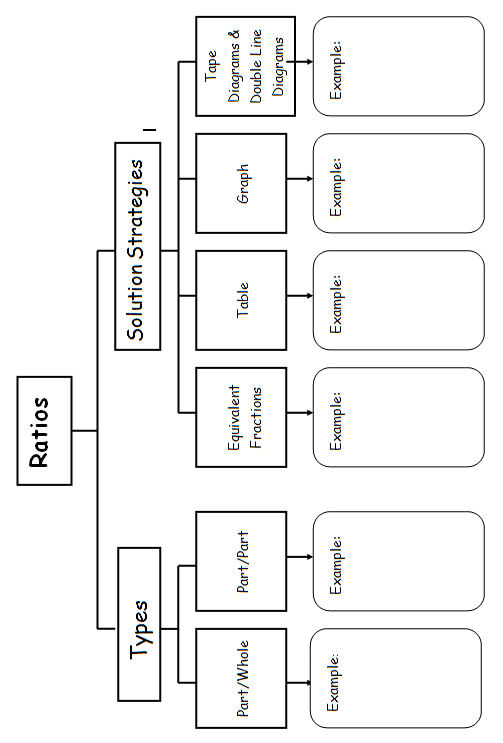
* 1. Check your answer with another method, and explain how this method justifies your original answer.

**Students can use a table or equivalent fractions.**

1. A runner and a bike rider leave Calder School at the same time. For every mile that the runner completes, the bike rider will complete 3 miles. If the runner completes 3 miles, how far will the bike rider travel? Show how you got your answer using two different methods. **9 miles**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Runner |  | | |  | | |  | | |
| Bike |  |  |  |  |  |  |  |  |  |

**Students can use a table or equivalent fractions.**



## Lesson 6: Ratios - Assessment

**Brief Overview of Lesson:**

Students review ratio reasoning and vocabulary, as well as different methods of representing and solving ratio problems. The review is followed by a summative assessment. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students understand ratios and ratio vocabulary, and can use multiple strategies to solve mathematical and real-life ratio problems.

**Estimated Time (minutes):** Two 50 min lessons

**Resources for Lesson:**

* Lesson 6: *In-Class Work: Ratio Review*
* Lesson 6: *Ratio Assessment*

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 6: Ratios - Assessment**

**Time (minutes):** Two 50-minute lessons

**By the end of this lesson students will know and be able to:**

* To use ratios/rates to solve mathematic or real-world problems.
* Demonstrate their knowledge of mathematical strategies for solving problems involving ratios, including tables, tape diagrams, double line diagrams, equivalent fractions, graphs, etc.
* Propose, justify, and communicate a solution (both orally and in writing).

**Essential Question(s) addressed in this lesson:**

* When is it useful to be able to relate one quantity to another?
* What strategies do I know for solving ratio problems?
* How do I communicate my solutions to others?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

**6.RP.3** Use ratio reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

**SMP.1** Make sense of problems and persevere in solving them. (Students must understand the problem context in order to translate into a ratio).

**SMP.2** Reason abstractly and quantitatively. (Students must understand the relationship between two quantities in order to express them mathematically.)

**SMP.3** Students construct viable arguments and critique the reasoning of others. (Students justify their solutions and look critically at others’ reasoning.)

**SMP.4** Model with mathematics. (Students model a real-life situation using mathematics.)

**SMP.7** Look for and make use of structure. (The structure of a ratio is unique and can be used across a wide variety of problem-solving situations.

**6.RST.4** Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

**Teacher Notes:**

The first day of this two-day lesson is a review.

Students are given a review sheet with the following types of problems. These are simple, uncomplicated examples designed to remind students of different concepts and strategies. Put one problem type in each of nine boxes.

Review Problem Types (Note: all problems require students to make sense of problems and persevere in solving them. (SMP.1)

1. Write ratios based on a shaded fraction picture, including part:part and part:whole ratios.
2. Write ratios based on a graphic of scattered objects, including part:part and part:whole ratios.
3. Translate between different forms of ratio notation.
4. Find equivalent fractions.
5. Fill in missing values on a data table of equivalent ratios.
6. Graph the proportional relationship of two quantities.
7. Use a graph to find missing values.
8. Use a tape diagram to solve a ratio problem.
9. Use a double line diagram to convert measurements.

On the second day of the two-day lesson, students take a summative assessment on ratio reasoning and problem-solving.

Scaffolding for at-risk students:

1. Vocabulary list with definitions
2. Table of measurement equivalencies
3. Use of notes and examples from previous day

**Lesson Sequence:**

Day 1:

1. Assign in-class ratio review. After working on the problem sheet individually, students compare their answers with a partner, and take turns explaining the solution to each other. Note: As students communicate their explanations they are also simultaneously engaging in SMP.3 (Construct viable arguments and critique the reasoning of others). Circulate among students to address misconceptions and to keep partnerships functioning smoothly.
2. Ask, “Which problems were the most difficult?”
3. Reveal the answers one-by-one on the overhead projector, reviewing any that were considered difficult, giving extra practice examples if needed, and noting common misconceptions observed.
4. Students take notes on their problem sheets, which will become their study guides.
5. Ask students to study for the next day’s test, including:
   * Reviewing the nine boxes of examples and notes done in class.
   * Making up extra practice problems for themselves or classmates, and solving them.
6. Day 1: Extended Learning/Practice (Homework): Review for the test, using notes and writing practice problems to solve.

**Day 2** Students take the assessment.

**Summative Assessment :** *Ratio Assessment*

**Preview outcomes for the next lesson:** Students will learn to solve problems about rates and unit rates.

**Lesson 6: In-Class Work: Ratio Review**

|  |  |  |
| --- | --- | --- |
| 1. **part: part part:whole**      * Draw a rectangle and divide it into 6 equal parts. Shade several parts. * What is the ratio of shaded parts to white parts? * What is the ratio of shaded parts to the total of the parts? | 1. **part: part part: whole**   three triangles, two squares, one circle   * Which shapes are in a ratio of 2:3? * What is the ratio of the triangles to the total number of shapes? | 1. **ratio notation**   three triangles, two squares, one circle   * Write the ratio of circles to triangles in these forms:   Colon: \_\_\_\_\_\_ : \_\_\_\_\_\_\_  Fraction:  Words: |
| 1. **equivalent ratios**   A cake recipe calls for 1 cup of sugar for every 4 cups of flour.   * How many cups of sugar are needed if you have 64 cups of flour?   **equivalent ratios-    1/4 and 2/8 are equal to how many 64ths?**   * How many cakes will that make? | 1. **data table of equivalent ratios**   There are 5 toes on one foot.  Fill in the missing values.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Feet | 4 |  |  | 6 | | Toes |  | 15 | 5 |  | | 1. **graphing the relationship between two quantities**   There are 2 pints in one quart.  Fill in the missing values.  a table showing pints and quart and a blank grid with the verical axis respresentint quarts and the horizontal axis representing pints. |

|  |  |  |
| --- | --- | --- |
| 1. **using a graph to find missing values**   Use the graph of pints to quarts.   * How many quarts equal 8 pints? * How many quarts equal 5 pints?   Mark your answers on the graph. | 1. **using a tape diagram**   Make a tape diagram to show the relationship of pints to quarts. | 1. **double line diagram**   Trent can bike 12 miles in one hour. He bikes for 6 hours one Saturday. Make a double line diagram that shows how many miles he will have biked after each hour. |

**Lesson 6: In-Class Work: Ratio Review ANSWER KEY**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. **part: part part: whole**      * Draw a rectangle and divide it into 6 equal parts. Shade several parts.  |  |  |  | | --- | --- | --- | |  |  |  | |  |  |  |  * What is the ratio of shaded parts to white parts?   **2:4**   * What is the ratio of shaded parts to the total of the parts? **2:6** | 1. **part: part part: whole**   Pictures of three triangles, two squares and one circle   * Which shapes are in a ratio of 2:3?   **Square: triangle**   * What is the ratio of the triangles to the total number of shapes?   **3:6 or 1:2** | 1. **ratio notation**   three triangles, two squares, one circle   * Write the ratio of circles to triangles in these forms:   Colon: \_\_\_1\_\_\_ : \_\_\_3\_\_\_  Fraction: 1/3  Words: 1 to 3 |
| 1. **equivalent ratios**   A cake recipe calls for 1 cup of sugar for every 4 cups of flour.   * How many cups of sugar are needed if you have 64 cups of flour?   **1/4 = 2/8 = how many 64ths**  **16 cups**   * How many cakes will that make?   **16 cakes** | 1. **data table of equivalent ratios**   There are 5 toes on one foot.  Fill in the missing values.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Feet | 4 | 3 | 1 | 6 | | Toes | 20 | 15 | 5 | 30 | | 1. **graphing the relationship between two quantities**   There are 2 pints in one quart.  Fill in the missing values. 10, 6, 2, 1   |  |  |  |  |  | | --- | --- | --- | --- | --- | | **Pints** |  | **4** |  | **2** | | **Quarts** | **5** |  | **3** |  |   graph verical axis quarts 0,2,4,6 horizontal axis pints 0,5,10,15 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. **using a graph to find missing values**   Use the graph of pints to quarts.   * How many quarts equal 8 pints? * How many quarts equal 5 pints?   **4 quarts**  **2 ½ pints**  Mark your answers on the graph. | 1. **using a tape diagram**   Make a tape diagram to show the relationship of pints to quarts.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Pints |  |  |  |  | | Quarts |  | |  | | | 1. **double line diagram**   Trent can bike 12 miles in one hour. He bikes for 6 hours one Saturday. Make a double line diagram that shows how many miles he will have biked after each hour.  a double line diagram showing hourse 1-5 and miles 1-72 (with a scale of 12). |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  |   Write a ratio for the shaded squares to the unshaded squares.  Write a ratio for the unshaded squares to the total number of squares. | 2.  15 shapes: three squares; four triangles; five circles; two trapezoids, one pentagonWhich shapes are in the ratio of 3:2?  5: 15 is the ratio for \_\_\_\_\_\_\_\_\_\_\_\_\_ to total shapes. | 3.  There are 30 birds to 3 cages. Write this ratio using the : form.  Each truck can carry 3 boxes. Write this ratio as a fraction.  Two trucks can carry 6 boxes. Write this ratio using the “to” form. |
| 4.  In a deck of 52 cards, one out of every 4 cards is a heart. How many cards are hearts?  In a deck of cards, there are 3 face cards (Jack, Queen, and King) in each of the 4 suits.  Express in fraction form the ratio of face cards that are diamonds to the total number of diamonds in the deck.  Express in fraction form the ratio of the total number of face cards in the deck to the total number of cards in the deck. | 5.  Mrs. Brown buys packages of pencils with 5 pencils per package. Fill in the chart below to show how many pencils she will get, depending on the number of packages that she buys.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Packages | **1** | **2** |  |  |  | **7** | | Pencils | **5** |  | **20** | **15** | **100** |  | | 6.  The perimeter of a square can be found by multiplying the side S x 4. Fill in the chart below.   |  |  | | --- | --- | | Side Length  (S) | Perimeter  (P) | | 1 |  | | 2 |  | | 3 |  | | 4 |  | | 5 |  | | 6 |  |   On a separate piece of paper, graph the side length (*x* axis) compared to the perimeter (*y* axis). Label the graph and axes. |

**Lesson 6: Ratio Assessment**

|  |  |  |
| --- | --- | --- |
| 7.  Using the graph that you made for Problem 6, answer the following questions:  A.  What is the perimeter of a square with a side of 3.5?  B.  If the perimeter is 21 units, what would be the length of one side?  C.  Extend the line on your graph. If the perimeter is 36 inches, what will be the length of the side?  D.  Mark and label your answers A, B, and C on your graph. | 8.  **Use a tape diagram** to answer this question:    One recipe makes a batch of 12 cookies. If you need to make 84 cookies, how many batches of cookies do you need to make? | 9.  Use a double line diagram to answer this question:  The speed of a runner is 5 meters every 2 seconds. How many meters will the runner go in 9 seconds?  Meters  5  |  |  2  Seconds |

**Lesson 6: Ratio Assessment**

**Lesson 6: Ratio Assessment ANSWER KEY**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  |   Write a ratio for the shaded squares to the unshaded squares.  **9:3**  Write a ratio for the unshaded squares to the total number of squares.  **3:12 or 1:4** | 2.  Which shapes are in the ratio of 3:2 ?  **squares to trapezoids**  15 shapes: three squares; four triangles; five circles; two trapezoids, one pentagon5: 15 is the ratio for **\_\_\_circles\_\_\_\_** to total shapes. | 3.  There are 30 birds to 3 cages. Write this ratio using the: form.  **30:3**  Each truck can carry 3 boxes. Write this ratio as a fraction.  **1/3**  Two trucks can carry 6 boxes. Write this ratio using the “to” form.  **2 to 6** |
| 4.  In a deck of 52 cards, one out of every 4 cards is a heart. How many cards are hearts?  **13 hearts**  In a deck of cards, there are 3 face cards (Jack, Queen, and King) in each of the 4 suits.  Express in fraction form the ratio of face cards that are diamonds to the total number of diamonds in the deck.  **3:52**  Express in fraction form the ratio of the total number of face cards in the deck to the total number of cards in the deck. **12:52** | 5.  Mrs. Brown buys packages of pencils with 5 pencils per package. Fill in the chart below to show how many pencils she will get, depending on the number of packages that she buys.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Packages | **1** | **2** | **4** | **3** | **20** | **7** | | Pencils | **5** | **10** | **20** | **15** | **100** | **35** | | 6.  The perimeter of a square can be found by multiplying the side S x 4. Fill in the chart below.   |  |  | | --- | --- | | Side Length  (S) | Perimeter  (P) | | 1 | **4** | | 2 | **8** | | 3 | **12** | | 4 | **16** | | 5 | **20** | | 6 | **24** |   On a separate piece of paper, graph the side length (*x* axis) compared to the perimeter (*y* axis). Label the graph and axes. |

**6: Ratio Assessment ANSWER KEY**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 7.  Using the graph that you made for Problem 6, answer the following questions:  A.  What is the perimeter of a square with a side of 3.5?  **14**  B.  If the perimeter is 21 units, what would be the length of one side?  **5.25 or 5 1/4**  C.  Extend the line on your graph. If the perimeter is 36 units, what will be the length of the side?  **9 units**  D.  Mark and label your answers A, B, and C on your graph. | 8.  **Use a tape diagram** to answer this question:    One recipe makes a batch of 12 cookies. If you need to make 84 cookies, how many batches of cookies do you need to make?  Batch   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |   Cookies | 9.  Use a double line diagram to answer this question:  The speed of a runner is 5 meters every 2 seconds. How many meters will the runner go in 9 seconds?  **a double line diagram comparing meters and seconds.**  **22 ½ or 22.5 meters** |

## Lesson 7: Understanding Rates and Unit Rates

**Brief Overview of Lesson:**

Students are introduced to rates and unit rates as special kinds of ratio. They use tools such as tape diagrams and double line diagrams to solve problems, using rate language such as *each, to, per, for each*, etc. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students understand ratios and how they express the relationship between two quantities.
* Students understand that division problems can be expressed in fraction form, e.g. ¾ = 3 ÷ 4 (6.NS.1)
* Students understand how to visually represent relationships using a tape diagram or a double line diagram.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

Lesson 7: *Formative Assessment: Rates*

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 7: Understanding Rates and Unit Rates**

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* A rate is a special kind of ratio that compares two types of measurements.
* A unit rate is the ratio of two measurements in which one of the measurements is a single unit.

*By the end of this lesson students will able to:*

* Find rates and unit rates using tools such as tables, tape diagrams, and double line diagrams; use rate language (*each, to, per, for each*, etc.).

**Essential Question(s) addressed in this lesson:**

* Why is it important to know how to use rates?
* How are rates used in everyday life?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar. “ “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

**6.RP.3**: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

**SMP.1:** Make sense of problems and persevere in solving them.

**SMP.7:** Look for and make use of structure.

**Teacher Notes:**

A **rate** is a measure, quantity, or frequency, typically one measured against a different type of quantity or measure:

* *A rate of speed of 60 miles an hour.*
* A measure of a part with respect to a whole; a proportion: *the mortality rate; a tax rate, the interest rate.*
* The cost per unit of a commodity or service: *postal rates.*

A **unit rate** is a ratio where a quantity of one measure is compared to a single unit of another:

* *miles per gallon, cost per pound, miles per hour, feet per second, heartbeats per minute, gallons per minute.*

Show students how to solve problems using **tape diagrams** and double number lines diagrams.

If 2 lbs. of beans cost $5, how much will 15 lbs. of beans cost?

Method 1:

I found that 14 pounds cost $35, and 1 pound is another $2.50, so that is $37.50 in all.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| pounds | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| dollars | 5 | 10 | 15 | 20 | 25 | 30 | 35 |

Method 2: I found that 1 pound costs $2.50, so I multiplied 15 lbs. x $2.50 and got $37.50.

**an example of a tape diagram comparing miles ane hours for the example: On a bicycle you can travel 20 miles in four hours. What are the unit rates in this situation, (the distance you can travel in one hour and the amount of time required to travel one mile? 
**

**Anticipated Student Preconceptions/Misconceptions:**

* Students may have difficulty understanding that rates compare two types of measurement quantities.

**Lesson Sequence:**

Students learn that a rate is a special kind of ratio. Through working the problems in this lesson students look for and making use of structure. (SMP.7) For example they explicitly become aware that rates compare two different quantities and a unit rate measures one unit of a quantity with a single unit of another.

Pre-assessment: Discussion:

**What is a rate?** A rate is a comparison of two quantities using different types of measures. *Ex. Miles to gallons; pounds to inches*

**What is a unit rate?** A unit rate is when one measure of a quantity is compared to a single unit of another. Give students examples of rates.

Write them on a poster. Ex. Miles per gallons (125 gallons using 5 gallons), ounces to money (15 ounces for $1.20), words per minute (175 words in 2 minutes).

1. Have students partner up. Challenge each partnership to find three rates to share with the class. Encourage them to use resources in the classroom.
2. As students share with the class, they can record their examples on a poster to be kept up in the classroom during the unit.
3. Present students with the following rate: 125 miles using 5 gallons
   * Ask students: “If I only had one gallon, how far could I drive?”
   * Students work with a partner.

Explain that this is a unit rate.

1. Ask students to convert rates to unit rates, using tape diagrams, double number lines, equivalent ratio, and division methods. (Students need extensive practice with visual models before moving to the division procedure shown below.)

125 gallons using 5 gallons.

a set of equivalent ratios representing miles per gallon.
125 miles/5 gallons equals x miles/one gallon..

1. Students write a journal entry: What is a rate and what is a unit rate?
2. Ticket to Leave: Describe a situation where knowing how to find a unit rate would be very important.
3. Extended Learning/Practice (homework): Students will bring two rates or unit rate examples from home/outside of school to school to share.

**Pre-Assessment:** Discussion to uncover real-world knowledge of rates

**Formative Assessment:**

Lesson 7: *Formative Assessment*

Ticket to Leave – See #6 above.

**Summative Assessment: None**

**Preview outcomes for the next lesson:** Students will continue to practice solving word problems/real-life situations involving rates and unit rates.

**Lesson 7: Formative Assessment: Rates**

A rate compares two quantities with different units.

1. There are 3 cars for 15 people. Write this as a rate. How many people per car?
2. The tree is 12 feet tall. The tree’s shadow is 6 yards long. Write this as a rate of tree height to shadow length.
3. Using the rate from Problem 2, make a chart of tree heights compared to shadow lengths for trees that are 1 foot tall, 3 feet tall, 6 feet tall, and 18 feet tall.
4. In a fruit basket, there are 2 apples for every 3 oranges.

To make a bigger fruit basket with 20 apples, how many oranges are needed?

**Lesson 7: Formative Assessment: Rates ANSWER KEY**

A rate compares two quantities with different units.

1. There are 3 cars for 15 people. Write this as a rate. How many people per car? **5 people**
2. The tree is 12 feet tall. The tree’s shadow is 6 yards long. Write this as a rate of tree height to shadow length.

**12:6 or 2:1 (1/2 yard per foot)**

1. Using the rate from Problem 2, make a chart of tree heights compared to shadow lengths for trees that are 1 foot tall, 3 feet tall, 6 feet tall, and 18 feet tall.

|  |  |
| --- | --- |
| **Tree (ft)** | **Shadow (yd)** |
| **1** | **.5** |
| **2** | **1** |
| **3** | **1.5** |
| **6** | **3** |
| **18** | **9** |

1. In a fruit basket, there are 2 apples for every 3 oranges.

To make a bigger fruit basket with 20 apples, how many oranges are needed?

**30 oranges**

## Lesson 8: Solving Problems with Unit Rates

**Brief Overview of Lesson:**

Students use mathematical strategies for solving problems, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, and graphs. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students understand that division problems can be expressed in fraction form, e.g. ¾ = 3 ÷ 4 (6.NS.1)
* Students understand how to visually represent relationships using a tape diagram or a double line diagram.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

None required

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 8: Solving Problems with Percents**

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* Problems-solvers use a variety of strategies to solve rate and unit rate problems.

*By the end of this lesson, students will be able to:*

* Determine when it is appropriate to use unit rates to solve mathematical or real life problems.
* Use mathematical strategies for solving problems involving unit rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.

**Essential Question(s) addressed in this lesson:**

* Why is it important to know how to use unit rates?
* How are unit rates used in everyday life?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

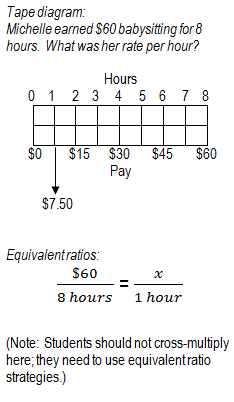
**6.RP.2**: Understand the concept of a unit rate a/b associated with a ratio a:b with b0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar. “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

**6.RP.3:** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

**SMP.2:**  Reason abstractly and quantitatively.

**Teacher Notes**

There are many different strategies for solving a unit rate problem:



*Table:*

|  |  |
| --- | --- |
| Hours Worked | Pay |
| 8 | $60 |
| 4 | $30 |
| 2 | $25 |
| 1 | ? |

*Division: $60 ÷ 8 = $7.50*

**Anticipated Student Preconceptions/Misconceptions:**

* Students may have difficulty understanding that rates compare quantities in different measures.
* Students maybe confused about using division to find a unit rate.
* Students may have difficulty labeling the different quantities properly.

**Lesson Sequence:**

Students learn that a unit rate is a ratio where a quantity of one measure is compared to a single unit of another: *miles per gallon, feet per second, heartbeats per minute, gallons per minute.*

1. Do Now/Warm Up: Joey is shopping for decks of cards. He finds a deal on cards that is on sale for a special rate of 5 packs for $5.95. At this rate, what is the cost of one pack of cards?

At another store, a deck of cards is 99¢ each. Which is the better deal?

1. Discuss:

* When is finding a unit rate important?
* Why do we make the choice to purchase what we do?
* Are we making the best decision? Is the purchase cost effective?
* What is important to know when purchasing items at the store?

1. List examples of unit rates found in everyday life.

* Miles per gallon
* Miles per hour
* Cost per pound
* Passengers per bus load
* Roses per bouquet
* Words per minute
* Pages per minute
* Heartbeats per minute
* Laps per hour/minute
* Dollars per item
* Feet per second
* Centimeters per inch

1. Students add unit rates from their homework to this list.
2. Review strategies for finding unit rates, including using tape diagrams, double number lines, equivalent ratios, and division method.
3. Guided Practice:

Solve this problem using tape diagram, equivalent ratios, and division so that students see that all three methods yield the same result. Note: By abstracting the problem via symbolic representation (tape measure) and then solving it, students are engaging in SMP.2 (Reason abstractly and quantitatively.).

Which would you choose?

Brand A: 1 box of 30 cookies for $3.00

Brand B: 2 boxes of 20 cookies each selling for $5.00 for 2 boxes

Find the cost per cookie for each brand. Which would you choose? Why? Show/explain your work.

*Extension: The cookie problem can also be solved by making the amount of money equal (for example, $15) and figuring out how many cookies you could buy for $15.*

*Or you could figure out the cost of the same number of cookies (for example, 120 cookies) and then compare the costs.*

1. Students practice solving rate and unit rate problems (not provided). Ask students to do some problems using two different strategies.

Circulate among the class, and choose several students who have particularly unique, efficient, or clever strategies for solving problems to show and explain their work to the class.

1. Extended Learning/Practice (homework)

Students solve a unit rate problem and write their own unit rate problem. They may use one of the rates from the class poster.

**Preview outcomes for the next lesson:** Students use problem-solving skills to apply strategies to mathematical and real-world problems.

## Lesson 9

## Using Rates and Unit Rates to Make Informed Consumer Decisions

**Brief Overview of Lesson:**

Students use rates and unit rates to solve real-world problems. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students know how to solve rate and unit rate problems using a variety of strategies.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Sam’s Market Weekly Sale Flyer
* Summative Assessment: Justin’s Pizza

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 9:** Using Rates and Unit Rates to Make Informed Consumer Decisions

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:* Knowledge of unit rates and rates can help consumers make informed purchasing decisions.

*By the end of this lesson students will be able to:*

* Determine when it is appropriate to use unit rates to solve mathematical or real life problems.
* Use mathematical strategies for solving problems involving unit rates, such as tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.

**Essential Question(s) addressed in this lesson:**

* Why is it important to know how to use rates and unit rates?
* How are rates and unit rates used in everyday life?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b0, and use rate language in the context of a ratio relationship, which is a rate of $5 per hamburger.”

**6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

**SMP.2**Reason abstractly and quantitatively

**ELA 6.RST.4** Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to Grades 6- 8 texts and topics.

**Teacher Note:** Some students may need guidance in knowing how to read a store flyer (layout, rate language, vocabulary, types of information).

**Anticipated Student Preconceptions/Misconceptions:**

Students may make impulsive decisions about which offer is the best deal based on the language of the offer rather than the cost per item or unit.

**Lesson Sequence:**

1. Warm-up/Do Now: Students exchange unit rate problems generated from the previous night’s homework and solve them.
2. Grocery store flyer scavenger hunt:
   * Students are provided with a worksheet that lists various grocery items (Sam’s Market Sale Flyer).
   * They find the unit rate for a single item.
   * Use this as a formative assessment. Not all students may be able to find the unit price.
3. Justin’s Pizza Assessment (problem-solving with unit rates)

Justin needs to buy four pizzas for a party.

He found the following three ads in his local newspaper:

|  |  |  |
| --- | --- | --- |
| **Rosa’s Pizza** | **Pizza Mart** | **Collins Pizza** |
| Each 12-inch pizza: $6.35  Buy 3 and get one free! | Each 12 inch pizza: $4.65 | Four-pack of  12-inch pizzas:  $19.60 |

Where would Justin get the best deal?

Answer: Pizza Mart ($18.60), Rosa’s Pizza ($19.05), Collins’ Pizza ($19.60)

Note: Look for clues on students’ reasoning abstractly and quantitatively (SMP.2) as they work the problems. For example the use of tools like tape measures to abstract the problem and represent it symbolically.

1. Extended Learning/Practice (homework)

Students research sales flyers, grocery, clothing, electronics, etc., and compare prices on two items. Flyers sources: newspapers, stores, internet, or provided by the school. They bring examples into class for sharing.

**Summative Assessment:** *Justin’s Pizza*

**Preview outcomes for the next lesson:**

* Students will be assessed on rates and unit rates.
* Students can use unit rates to explain and make informed decisions about purchases.

Sam’s Market Weekly Sale Flyer

Here are some items on sale at Sam’s Market. Find the unit rate (unit price) of each item, and fill in the chart below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ITEM | Quantity Listed | Price Listed | Unit | Unit Price |
| Oranges | 12 | $3.60 | 1 orange |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**Sam’s Weekly Market Flyer ANSWER KEY**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ITEM | Quantity Listed | Price Listed | Unit | Unit Price |
| Oranges | 12 | $3.60 | 1 orange | $.30 |
| Water | 24 | $3.60 | 1 bottle | $.15 |
| Snack pack | 5 | $5.50 | 1 bag | $1.10 |
| Sauce | 3 | $9.69 | 1 jar | $3.23 |
| Milk | 2 | $5.98 | 1 gal | $2.99 |

**Justin’s Pizza**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Justin needs to buy four pizzas for a party.

He found the following three ads in his local newspaper:

|  |  |  |
| --- | --- | --- |
| **Rosa’s Pizza** | **Pizza Mart** | **Collins Pizza** |
| Each 12-inch pizza: $6.35  Buy 3 and get one free! | Each 12 inch pizza: $4.65 | Four-pack of  12-inch pizzas:  $19.60 |

Where would Justin get the best deal?

**Justin’s Pizza ANSWER KEY**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Justin needs to buy four pizzas for a party.

He found the following three ads in his local newspaper:

|  |  |  |
| --- | --- | --- |
| **Rosa’s Pizza** | **Pizza Mart** | **Collins Pizza** |
| Each 12-inch pizza: $6.35  Buy 3 and get one free! | Each 12 inch pizza: $4.65 | Four-pack of  12-inch pizzas:  $19.60 |

Where would Justin get the best deal?

**Rosa’s Pizza: $19.05**

**Pizza Mart: $18.60**

**Collins Pizza: $19.60**

## Lesson 10: Rates and Unit Rates Assessment

**Brief Overview of Lesson:**

Students are assessed on their knowledge of rates and unit rates through a problem-solving situation requiring students to compare costs and unit prices at two different stores. They make a recommendation for which store has the best values, and defend their choice in writing, using computations and ratio/rate reasoning as evidence. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students understand that ratios relate two quantities to one another.
* They understand that rates are special kinds of ratios that relate two different measurement quantities to each other.
* Given a rate, students know how to find a unit rate.
* Students understand that another word for “flyer” is “circular”, and that these circulars advertise special money-saving deals at stores.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Little Red Riding Hood’s Grocery Trip

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics, Grade 8**

**Lesson 10:** Rates and Unit Rates Assessment

**Time (minutes):** 50

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:* Several strategies for solving rate and unit rate problems.

*By the end of this lesson, students will be able to:*

* Solve real world and mathematical rate/unit rate problems.
* Use rate language to describe the relationship between two quantities. (each, to, per, for, /, each, etc.)

**Essential Question(s) addressed in this lesson:**

* Why is it important to know how to use ratios and rates?
* How are ratios and rates used in everyday life?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar. “ “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

**6.RP.3**Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

**SMP.1**Make sense of problems and persevere in solving them.

**SMP.2** Reason abstractly and quantitatively.

**SMP.4** Model with mathematics

**Anticipated Student Preconceptions/Misconceptions:**

* Students may have trouble distinguishing between rates and unit rates.
* Students may find it difficult to read and interpret an authentic grocery store flyer that uses rates and unit rates.

**Lesson Sequence:**

1. Warm Up/Do Now: In this week’s grocery circular, Giant foods advertises bananas on sale for 5 lbs. for $4.45. Food Mart has them on sage for $.95/lb. Which store has the best deal? (*Giant Foods: $.89/lb*)
2. Briefly review different strategies for solving rate/unit rate problems:
   * Equivalent ratios
   * Equations, including division
   * Tape diagrams and double line diagrams
3. Assessment: *Little Red Riding Hood’s Grocery Trip*

Students are given two grocery store circulars. They are asked to calculate the unit rate on a variety of items. They will use the unit rates to compare prices to determine the store with the best overall “best value”.

1. Extended Learning/Practice (homework)

To prepare for the next lesson, students find examples of percents at home and bring in three examples of percents (including their contexts). Possible sources include newspapers, magazines, store flyers, bills, and internet sources.

**Summative Assessment: Little Red Riding Hood’s Grocery Trip**

**Preview outcomes for the next lesson:**

Students study percents.

**Lesson 10: Assessment**

**Little Red Riding Hood’s Grocery Trip**

After spending some time phoning and texting with Granny, Little Red Riding Hood decided it was time to visit her beloved grandmother, who lived in the Far, Far Away Woods. She thought it would be nice if she brought some groceries and fixed lunch for her elderly, yet hip, Granny. She remembered that her Granny loved tacos and brownies, so she decided that she would make them for her as a surprise! She quickly made her grocery list. Red decided she needed to buy:

* soft taco shells
* shredded cheese
* ground beef
* salsa
* lettuce
* brownies (for dessert, of course!)

Little Red Riding Hood scanned recent grocery store flyers and found that two different stores, the Very Hungry Wolf’s Food Store and the Strong, Brave Woodsman’s Food Mart, had all of the ingredients that she needed. Red didn’t have a lot of time to shop, so she decided to use the flyers to compare prices to help her decide which store had the best values. After comparing unit prices for each item, she totaled the unit prices and chose the store that would give her the best overall value.

**Determine the unit price** for each item at each grocery store. Put your answers in the place provided in the chart shown. You must show your work for each item. Once you have decided which grocery store will offer Red the best value for her money, explain why she should shop there. Give specific detail about how you decided whether she should shop at the Very Hungry Wolf’s Food Mart or the Strong, Brave Woodsman’s.

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| --- | --- | --- | --- | --- | --- | --- |
| **The Very Hungry Wolf’s Food Store** | | |  | **The Brave Woodsman’s Food Mart** | | |
| **Grocery Item** | **Price** | **Unit Price** | Grocery Item | Price | Unit Price |
| Soft Taco Shells | 8 for $.96 |  | Soft Taco Shells | 12 for $1.68 |  |
| Shredded Cheese | 7 oz. for $1.75 |  | Shredded Cheese | 1 lb. for $3.20 |  |
| Ground Beef | 1 lb. for $2.50 |  | Ground Beef | 5 lbs. for $9.95 |  |
| Salsa | 22 oz. for $3.30 |  | Salsa | 10 oz. for $1.98 |  |
| Lettuce | 1 head for $.89 |  | Lettuce | 3 heads for $1.69 |  |
| Brownies | 12 for $3.60 |  | Brownies | 4 for $1.00 |  |

**Lesson 10: Assessment ANSWER KEY**

**Little Red Riding Hood’s Grocery Trip**

After spending some time phoning and texting with Granny, Little Red Riding Hood decided it was time to visit her beloved grandmother, who lived in the Far, Far Away Woods. She thought it would be nice if she brought some groceries and fixed lunch for her elderly, yet hip, Granny. She remembered that her Granny loved tacos and brownies, so she decided that she would make them for her as a surprise! She quickly made her grocery list. Red decided she needed to buy:

* soft taco shells
* shredded cheese
* ground beef
* salsa
* lettuce
* brownies (for dessert, of course!)

Little Red Riding Hood scanned recent grocery store flyers and found that two different stores, the Very Hungry Wolf’s Food Store and the Strong, Brave Woodsman’s Food Mart, had all of the ingredients that she needed. Red didn’t have a lot of time to shop, so she decided to use the flyers to compare prices to help her decide which store had the best values. After comparing unit prices for each item, she totaled the unit prices and chose the store that would give her the best overall value.

**Determine the unit price** for each item at each grocery store. Put your answers in the place provided in the chart shown. You must show your work for each item. Once you have decided which grocery store will offer Red the best value for her money, explain why she should shop there. Give specific detail about how you decided whether she should shop at the Very Hungry Wolf’s Food Mart or the Strong, Brave Woodsman’s.

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| --- | --- | --- | --- | --- | --- | --- |
| **The Very Hungry Wolf’s Food Store** | | |  | **The Brave Woodsman’s Food Mart** | | |
| **Grocery Item** | **Price** | **Unit Price** | Grocery Item | Price | Unit Price |
| Soft Taco Shells | 8 for $.96 | **$.12/each** | Soft Taco Shells | 12 for $1.68 | $.14/each |
| Shredded Cheese | 7 oz. for $1.75 | $.25/oz | Shredded Cheese | 1 lb. for $3.20 | **$.20/oz** |
| Ground Beef | 1 lb. for $2.50 | $.16/oz | Ground Beef | 5 lbs. for $9.95 | **$.12/oz** |
| Salsa | 22 oz. for $3.30 | **$.15/oz** | Salsa | 10 oz. for $1.98 | $.20/oz |
| Lettuce | 1 head for $.89 | $.89/each | Lettuce | 3 heads for $1.69 | **$.56/each** |
| Brownies | 12 for $3.60 | $.30/each | Brownies | 4 for $1.00 | **$.25/each** |

**Little Red Riding Hood should shop at the Brave Woodsman’s Food Mart, where 4 out of the 6 items cost less.**

## Lesson 11: What is a Percent?

**Brief Overview of Lesson:**

Students learn that percents are another special kind of ratio, where a quantity is compared to 100. They use 10 x 10 grids to graphically represent percent quantities. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students will have had experiences with fractions and equivalent ratios that are compatible with decimal tenths and hundredths. (½, ¼, ⅕, 1/10, 1/25, 1/100)

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

**Websites**

* Lesson 11 *Homework*
* *Battle of Fractions, Decimals, and Percents*
* Grids for *Battle of Fractions, Decimals, and Percents*
* 10 x 10 Grids
* 100 Chart

**Unit: Ratios, Rates, and Proportions**

**Content Area/Course: Mathematics, Grade 8**

**Lesson 11:** What is a Percent?

**Time (minutes):** 50

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* Ratios can be represented in a variety of formats, including percents.
* A percent is a type of ratio that compares a quantity to 100.
* Percents can be represented graphically in several ways.

*By the end of this lesson students will be able to:* Describe a ratio as a percent.

**Essential Question(s) addressed in this lesson:**

* What is a percent?
* How do percents relate one quantity to another?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6 RP.3c** Find the percent of a quantity as a rate per 100 (e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent

**6.SMP.7** Look for and make use of structure.

**Teacher Notes**

(Note: The pre-assessment [pizza question] is intended to be a quick dipstick of students’ prior knowledge. If students cannot answer questions, move on to the lesson sequence rather than making the pizza question the focus of the day’s lesson.)

The concept of percent first appears in Grade 6 in the new standards. Students have had extensive work in fractions and decimals, which will help them to find equivalencies in percents.

Percents are part/whole ratios where the whole is measured in hundredths.

Beginning students can derive rates per 100 from fractions that have denominators of 2, 4, 5, 10, 20, 25, and 50, by using their knowledge of fraction equivalencies.

Significant time should be spent on visual models such as 10 x 10 grids (this lesson), tape diagrams, and double-line diagrams (subsequent lessons) to help students see the comparison between a quantity and 100.

Students should be familiar with the following fractions ½, ¼, ⅕, 1/10, and 1/100.

Other useful manipulatives include base-ten blocks (hands-on or virtual), pennies, tiles, etc.

NCTM Illuminations *Grid and Percent It* Lesson has good background for teachers.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L249>

This resource presents a method of solving percent problems that focuses on the basic concept of percent, that of "parts per hundred." A 10 × 10 grid, which is a common model for visualizing percents, is extended to solve various types of percent problems. The activities in this resource utilizes the use of grids to help student understand and use percentages SMP.7 (Look for and make use of structure).

Virtual Manipulative Percent grids:<http://nlvm.usu.edu/en/nav/frames_asid_>333\_g\_3\_t\_1.html?from category\_g\_3\_t\_1.html

Students represent, name, and explore percentages using hundreds grids.

*Students who need more challenge would be ready to solve more difficult problems independently shading 6.5%, 33.3%,*

**Anticipated Student Preconceptions/Misconceptions:**

* Students may have difficulty understanding that 100% = the whole.
* Students may use the numerator of any fraction as the percent.

**Lesson Sequence:**

1. Discuss: What does “percent” mean? Students may have some real-world experience with percents.

Pre-assessment: Draw a sketch of a pizza with 8 slices. How much is 100% of the pizza? 50%? 25%?

1. In a small group, students share percents from Lesson 10’s homework (finding examples of percents at home) . They discuss what each percent might mean in its context. Examples can be posted in the classroom.
2. Discuss: When you are at a store and there is a sale that is 20% off, what does this mean?
3. Give each student a grid worksheet with 10 x 10 grids. Have students shade 10 x 10 grids in various amounts. Examples: 30%, 54%, 3%. Have students write each percent as a decimal and as a fraction: 54% = .54 = 54/100

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Example: This grid shows 25%.

Another way to show 25%:

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1. Partner work. Each student shades an amount on a grid. The other student then reads the amount shaded, giving the decimal, the fraction, and the percent names. Repeat with different examples. Challenge students to find as many different ways as possible to show a percent. For example, 20% can be shown as 2 x 10, 4 x 5, or 5 x 4 grids.
2. Require students to use models and drawings to explain their thinking for the following problems:

* The paper carrier delivers 100 newspapers every Sunday. If 75% were delivered before noon, how many newspapers still need to be delivered? (Extension: What if the paper carrier delivers 200 newspapers?)
* 25% of the cars at a car dealership come with a 5-year warranty. The rest have a 2-year warranty. How many have a 2-year warranty?

Sample representation:

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| 5 yr | 5 yr | 5 yr | 5 yr | 5 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr |
| 5 yr | 5 yr | 5 yr | 5 yr | 5 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr |
| 5 yr | 5 yr | 5 yr | 5 yr | 5 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr |
| 5 yr | 5 yr | 5 yr | 5 yr | 5 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr |
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| 2 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr | 2 yr |

* The math class has completed 80 pages of the 100 page geometry unit. What percent have they completed and what percent is left to do? Show on a hundreds grid.
* During the first 100 days of school there were 65 sunny days. What percent of days were not sunny?
* There were 100 pounds of potatoes harvested from the community garden.
  + 20% were sweet potatoes
  + 45 pounds were russet potatoes
  + The rest were golden potatoes

On a 10 x 10 grid represent each type of potato by both pound and percent. Explain how you found the number of pounds and the percent of golden potatoes.

1. Battle of Fractions, Decimals, and Percents Game (see below).
2. Return to the opening problem: If an item costs $100 but there is a 20% discount, how much will you pay?
3. Extended Learning/Practice (homework):

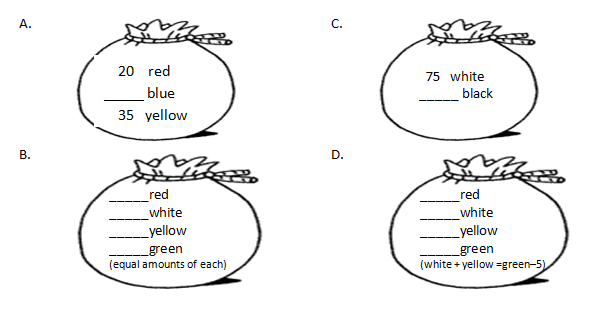
Worksheet: Percent problems (where the whole = 100).

**Preview outcomes for the next lesson:** Students expand their understanding of percents by representing any ratio as a quantity out of 100.

Example: 4/5 = 80/100 = 80%

**Lesson 11: Homework**

1. Each bag of marbles contains 100 marbles. Each bag below shows the *percents* of marbles for each color of in it. Find the missing *percent*s for each bag.



1. There are 100 students in sixth grade. Students in sixth grade can participate in one after-school music program. 22% chose chorus, 55% chose band, and 10% chose orchestra. What percent of sixth graders are not participating in a music program? Show your answer on a 10 x 10 grid.
2. There are 100 songs on Jed’s iPod. 70 songs are rock music, 8 songs are rap music, and 10 are pop music. The rest of the songs are country music. How many songs are country music? How do you know? What percent of the total songs is this?

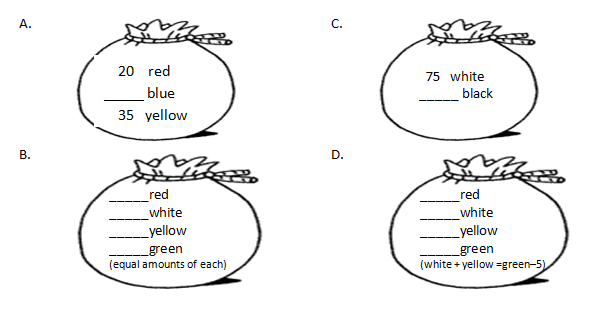
Challenge: If you could only put 100 songs on an iPod, which types of music would you choose? List the categories and that percent of songs in each type of music that you would choose.

1. A bucket contains 100 blocks. 30% of the blocks are yellow. 15% are red, 12% are blue, and 8% are orange. Show this on a 10 x 10 grid. Write each quantity as a fraction. Which colors could be added to total 50%? Which colors could be added to total 20%?

**Lesson 11: Homework ANSWER KEY**

Each bag of marbles contains 100 marbles. Each bag below shows the percentage of each color of marble of in it. Find the missing percents for each bag.

**A. 45% blue; B. 25% red, 25% white, 25% yellow, 25% green; C. 25% black; D. 35% red, 20% white, 20% yellow, 25% green**



1. There are 100 students in sixth grade. Students in sixth grade can participate in one after-school music program. 22% chose chorus, 55% chose band, and 10% chose orchestra. What percent of sixth graders are not participating in a music program? Show your answer on a 10 x 10 grid. **13%**
2. There are 100 songs on Jed’s iPod. 70 songs are rock music, 8 songs are rap music, and 10 are pop music. The rest of the songs are country music. How many songs are country music? How do you know? What percent of the total songs is this? **12 songs, 12%**

Challenge: If you could only put 100 songs on an iPod, which types of music would you choose? List the categories and that percent of songs in each type of music that you would choose. **Answers will vary.**

1. A bucket contains 100 blocks. 30% of the blocks are yellow. 15% are red, 12% are blue, and 8% are orange. Show this on a 10 x 10 grid. Write each quantity as a fraction. Which colors could be added to total 50%? Which colors could be added to total 20%?

**50 % = yellow + blue + orange**

**20 % = blue + orange**

**Lesson 11: Game**

**Battle of FRACTIONS, DECIMALS, and PERCENTS**

Object of the game:

Practice comparing fractions, decimals, and percents.

Number of Players: 2, 3,

Directions:

1. Distribute the cards equally to each player.
2. Players keep their cards face-down.
3. Each player turns over one card.
4. The player who has the card with the greatest value takes all cards.
5. If there is a tie, the players with the two highest cards place three additional cards face-down. Then they turn one more card face-up. The player who has the card with the greatest value takes all the cards.

OPTION 1:

The game continues in this way until all the cards have been played.

The player with the greatest number of cards wins the game.

OPTION 2:

When a player runs out of cards, he or she is eliminated from the game.

The game continues until one player has won all the cards.

Note: If students do not agree on the card with the highest value, have them use classroom resources (visual models, manipulatives, calculator) to determine the winner of the round.

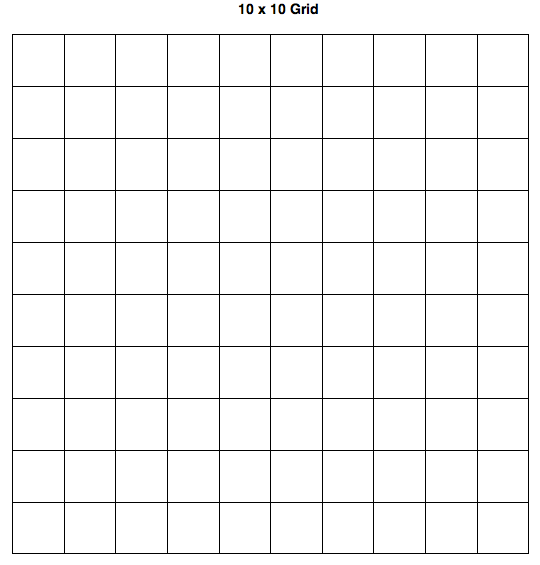
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| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **25%** | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.75** |  | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.1** |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **7/10** |  |  | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **1/2** |  |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **70%** | |

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| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **1/10** |  |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **75%** | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **10%** | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.7** |  | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.5** |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **1/5** |  |  | |

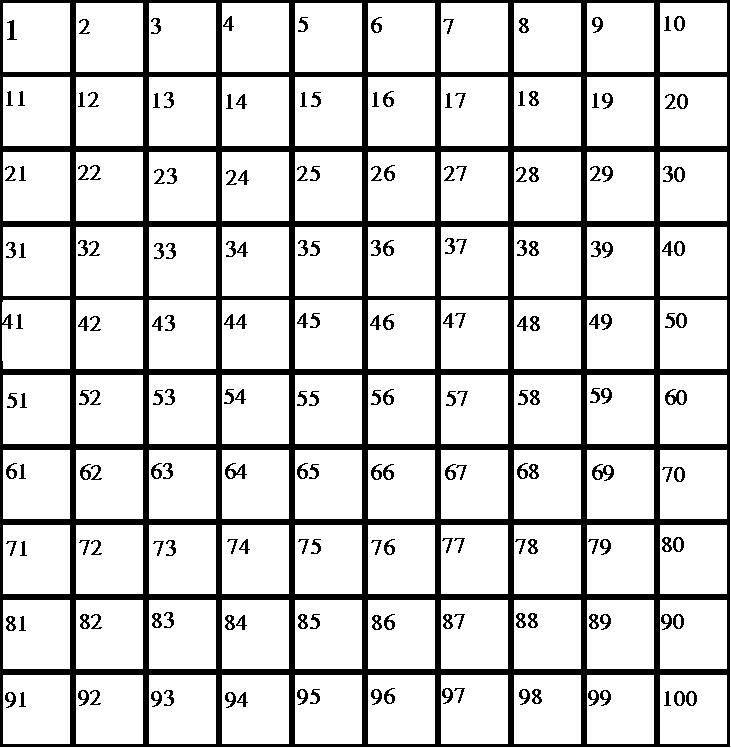
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| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **1/8** |  |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **80%** | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **12.5%** | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.6** |  | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.3** |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **6/10** |  |  | |

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| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  | **0.125** |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **3/5** |  |  | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **3/10** |  |  | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | | **1/3** |  |  | |
| |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **30%** | | |  |  |  | | --- | --- | --- | | Fraction | Decimal | Percent | |  |  | **30%** | |

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## Lesson 12: Percents and Tape Diagrams

**Brief Overview of Lesson:**

Students use tape diagrams to solve percent problems. They find the percent of a whole. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students understand how to find a percent when the whole is 100.
* Students can find equivalent fractions.
* Students are comfortable using tape diagrams to compare quantities.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Tape Diagrams
* Double Line Diagrams

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics, Grade 6**

**Lesson 12:** Percents and Tape Diagrams

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* The quantity represented by a percent depends upon the size of the whole.
* Visual representations of percents are useful in solving percent problems.

*By the end of this lesson, students will be able to:*

* Represent a percent of a number using tape diagrams.
* Compare a percent of different wholes.
* Write a statement in the form of \_\_\_% of \_\_\_\_ = \_\_\_\_\_

**Essential Question(s) addressed in this lesson:**

How can we represent percents visually to help us solve problems?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

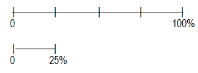
**6 RP.3c** Find the percent of a quantity as a rate per 100 (e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent

**6.SMP.7** Look for and make use of structure.

**Teacher Notes**

Students should access their prior knowledge of number lines and tape diagrams to represent fractions. (See previous lessons.)

Double line segments can be used to show the whole (100%) compared to a part.



Note: Tape measure and double number lines help students see patterns and structures (SMP.7: Look for and make use of structure).

If 75% of the budget is $1200, what is the total budget?

Dollars

$0 $1200 ?

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

0% 25% 50% 75% 100%

Remind students that tape diagram segment divisions will vary depending upon the percent. (Example: To find 20%, it is useful to divide 100 into 5 segments; to find 10%, it is useful to divide 100 into 10 segments, etc.)

Many students may be ready to use more challenging numbers such as 240, 175, and 350 as the whole for #4.

Advanced students might use the *Now and Then* PowerPoint and lesson to work more independently and figure out how prices change. They need a good understanding of percent to do these activities. <http://illuminations.nctm.org/LessonDetail.aspx?id=L837>

**Anticipated Student Preconceptions/Misconceptions:**

* Students may have difficulty understanding that the whole can be more than 100 when representing a percent.
* Students may have difficulty determining accurate and appropriate segments on tape diagrams.

**Lesson Sequence:**

1. Discuss with a partner or group: How could a 10 x 10 square grid represent 200 people? Be prepared to share your ideas. (Each square equals 2 people.)

Discuss with a partner or group: How could a tape diagram represent 200 people?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Percent: | 25% | 25% | 25% | 25% |
| People: | 50 | 50 | 50 | 50 |

Discuss the similarities and differences between the two visual models.

1. Students shade in 25% on tape diagrams where the whole represents different amounts. The amounts are: 100, 200, 60, 20, and 500. Challenge: 50, 250, 2.

Record the fractions represented in each tape diagram. What pattern can you see? What conclusion can be drawn?

* All fractions are equivalent and equal to 25% or ¼.
* 25% represents different quantities (depending upon the amount of the whole).

1. Students express the quantity in each tape diagram as follows:

25% of \_\_\_\_\_\_ = \_\_\_\_\_\_\_\_

(25% of 100 = 25; 25% of 500 = 125, etc.)

1. Students are given another percent (such as 10%, 20%, 50%, 75%, etc.) and are asked to create their own set of five tape diagrams in which they find the assigned percent of the following wholes: 100, 200, 60, 20, and 500. Record the fractions represented in each tape diagram in Step 4. What pattern do they see? What conclusion can be drawn?

Students express the quantity in each tape diagram as follows:

\_\_\_\_\_\_% of \_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ for each tape diagram.

1. Students share their tape diagrams, and conclusions.
2. Ticket to leave: Explain why 25% does not always represent the same amount.

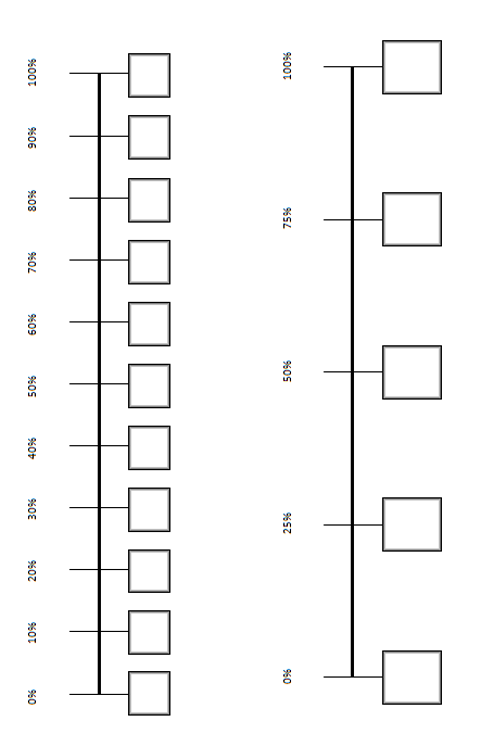
**Preview outcomes for the next lesson:** Students will represent solutions to percent problems with equations in addition to visual representations.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **10%** | **20%** | **30%** | **40%** | **50%** | **60%** | **70%** | **80%** | **90%** | **100%** |
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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **10%** | **20%** | **30%** | **40%** | **50%** | **60%** | **70%** | **80%** | **90%** | **100%** |
|  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **25%** | **50%** | **75%** | **100%** |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **25%** | **50%** | **75%** | **100%** |
|  |  |  |  |



## Lesson 13: Solving Percent Problems: Missing Part, Missing Percent

**Brief Overview of Lesson:**

Students learn that in percent problems, there are three quantities. Either the percent, the part, or the whole can be the unknown quantity. They solve for the unknown part or percent, (given the other two values). As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students see the connections between ratios and percents.
* Students can solve for a missing quantity in equivalent ratios.

**Estimated Time (minutes):** 50 mins

**Resources for Lesson:**

* Websites

**Unit: Ratios, Rates, and Proportions**

**Content Area/Course: Mathematics, Grade 8**

**Lesson 13:** Solving Percent Problems: Missing Part, Missing Percent

**Time (minutes):** 50 mins

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:* Percent problems contain three components: the percent, the part and the whole.

*Students will be able to:*

* Solve problems where they must find the unknown part or percent (given the other two values).
* Write equations to match percent problem contexts.

**Essential Question(s) addressed in this lesson:**

* How are percents used to solve problems?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6 RP.3c** Find the percent of a quantity as a rate per 100 (e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent.

**6.SMP.2** Reason abstractly and quantitatively.

**Teacher Notes**

Refrain from teaching arbitrary rules such as “*To determine 10% of a number, move the decimal point over one place to the left*” or procedures such as cross-multiplying to solve ratio equivalencies. These shortcuts will be part of Grade 7.

Specifically use the terms part, whole and percent during classroom discussions.Make connections between prior knowledge of common fractions and percents:

|  |  |
| --- | --- |
| Prior Knowledge | Connection to % |
| ¾ of 20 = 15 | 75% of 20 = 15 |
| 15 out of 20 = 3/4 | 15 out of 20 = 75% |
| ¾ = 75% | |

Encourage the use of mental math strategies and number sense to estimate solutions and check answers. For example, 40% of 200 must be less than half of 200.

**Anticipated Student Preconceptions/Misconceptions:**

* Students may have difficulty understanding a problem context in order to determine the missing value.
* Students may invert values when setting up equivalent ratios.

**Lesson Sequence:**

Solving problems to find the percent

1. Warm-up problems: Students work independently to solve the following problems, using any strategy that they choose. They share solutions in a small group.

* On Saturday, the bicycle tour covered 100 miles. I took my first break at 10 miles. What percent of the day’s ride was complete?
* The next day, the bicycle tour covered 50 miles. I stopped after 10 miles. What percent of that day’s ride was complete?
* If 50 students went to the cafeteria and 30 purchased chicken fingers, what percent of students purchased chicken fingers? (Scaffold: Change the fraction 30/50 to 60/100 by using equivalent fractions. Fill this in on the 10 by 10 grid. One student will be represented by 2 squares.)

1. While working the problems in this lesson student also engage in SMP.3 (Reason abstractly and quantitatively). For example in problem A, when the student creates a coherent representation (the grid) to represent the problem situation he/she is decontextualizing the problem situation.

Working with partners, solve:

* 1. Mrs. Brown purchased a new $800 computer for 10% off. What was the savings?

Students use a 10 x 10 grid to explain their thinking.

Ask students to represent the same problem using a tape diagram. Ask students to represent this problem with an equation of equivalent ratios.

This can be understood as: 10% is \_\_\_\_\_ out of 800. Students should also write the relationship in this way: 10% of $800 = $80

* 1. Mrs. Brown went to a different store. The same model of computer is usually priced at $1,000 but it is on sale for 25% off. How many dollars would she save? Ask students: Which strategy is best, a tape diagram or a 10 x 10 grid?

This can be understood as: 25% is \_\_\_\_\_ out of 1000.

* 1. From which store should Mrs. Brown purchase the computer?

Solving problems to find the missing percent

3.

* 1. Rose read 60 pages of 300 page book. What percent had she read? What percent does she still have to read?

Have students set up this equivalent ratio and solve using their knowledge of equivalent fractions. Justify the answer using a tape diagram.

This can be understood as: 60 out of 300 is \_\_\_\_%.

* 1. Robert read 20 pages of a 300-page book. What percent has he read? What percent does he still have to read? Explain how this problem was solved. (Calculators allowed.)
  2. Mill River Middle School has 320 families with students at the school. For the Open House, 240 families were represented. What percent of families came to Open House? Students solve this problem independently and then pair with another student to compare results and strategies.

* 1. Students write two percent problems (one with a missing part, and one with a missing percent). They solve their own problems, and then exchange problems with a friend.

4. Extended Learning/ Practice (homework): Students write two additional percent problems about missing part, missing percent.

**Preview outcomes for the next lesson:**

Students will learn how to find the missing whole, given the part and the percent.

## Lesson 14: Solving Percent Problems: Missing Whole

**Brief Overview of Lesson:**

Students extend percent problem-solving so that given the part and the percent, they can solve for the whole. They complete a summative assessment on percents. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students will know that a percent problem contains three parts: the percent, the part and the whole.
* Given the part and the whole, students can determine the percent.
* Given the percent and the whole, students can determine the part.

**Estimated Time (minutes):** Two 50-minute lessons

**Resources for Lesson:**

* Lesson 14: *Percent-Solving Worksheet*
* Summative Assessment: *Tim’s Tires, Raffle Ticket Sales, Library Books*

**Unit: Ratios, Rates, and Percents**

**Content Area/Course: Mathematics Grade 8**

**Lesson 14:** Solving Percent Problems: Missing Whole

**Time (minutes):** 2 50-minute lessons

**By the end of this lesson students will know and be able to:**

*By the end of this lesson students will know:*

* Percent problems have three parts: the part, the percent, or the whole. Given two parts, the student can solve for the missing part.
* Percents can be used to solve a variety of mathematical and real-world problems.

*By the end of this lesson students will be able to:*

* Determine the whole, given the part and the percent.
* Solve problems involving percents when the part, the percent, or the whole is the unknown quantity.
* Make decisions or judgments using their knowledge of percents.

**Essential Question(s) addressed in this lesson:**

* What types of problems can we solve with percents?
* How does knowing about percents help to make us to make decisions?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**6 RP.3c** Find percent of a quantity as a rate per 100(e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent

**6.SMP.1** Make sense of problems and persevere in solving them.

**Teacher Notes**

* Remind students that “percent” is not a new concept. It is a new notation for what they already know … fractions and decimals.
* Students should be encouraged to use their number sense and percent benchmarks to judge the reasonableness of answers. (i.e. greater than or less than 50%?)
* It is important for students to continue to draw a visual model to represent the problem, so a deeper conceptual understanding is developed.
* Make sure that students understand the meaning of key vocabulary in the word problems.
* Extension: Students conduct a survey and use the results to determine percents. These percents can then be used to construct a circle graph.

Students write a summary of the responses.

**Anticipated Student Preconceptions/Misconceptions:**

* Students may know that 100% represents the whole, but many students may still experience difficulties understanding that this whole can be any number (100, greater than 100, or less than 100).
* When setting up equivalent ratios, students may place quantities in the incorrect position.

**Lesson Sequence:**

Note: This sequence of lessons and assessments takes two class periods.

1. Students review the previous night’s homework by asking a friend to solve the two problems that they wrote.
2. Introduce finding the missing whole with the following problem:

*Yesterday, I bought a sweatshirt on sale for 50% off and ended up paying only $15. I also bought a t-shirt that was on sale for 25% off and ended up paying only $15. What was the original price for each?*

Using a quick “think-pair-share” protocol, ask students to determine the original prices, and share their answers and strategies with classmates.

1. Summarize percent problem solving strategies:

* Number sense (1/2 of something is $15, so the whole must be $30)
* Tape diagrams
* Equivalent ratios: 

Guide students through the solution for the following problem:

36% of the students at Central Middle School are sixth graders.   
 This is 180 students.

How many total students are enrolled at Central Middle School?

* What percent represents the total enrollment?
* What do we need to find out in this problem?
* What percent do we know? Is this the part or the whole?
* If 180 students or 36% are 6th graders, have students estimate the total number. (Hint: 36% is close to 1/3)
* Model the problem with a tape diagram.
* Solve the problem with equivalent ratios.
* Discuss: does this answer make sense? Can anyone suggest another strategy to solve this problem?

1. Assign the following problem:

60% of the people at the movie theater were adults, and the rest were children. There were 720 adults. How many people were at the movie theater in all?

Students work in pairs or small groups to solve the problem using two different strategies. Record work on chart paper.

Conduct a gallery walk to compare and comment on the approaches of each group.

1. Students identify the missing quantity in each of the following three problems: (part, percent, whole).

* Harry has 350 songs on his MP3 player. 15% are country, 35% are rock, and the rest are pop. How many songs are pop?
* There are 150 students at Whittier school who have been to New York City. This is 30% of the school’s total population. How many students attend Whittier School?
* Sally had 25 pairs of socks. Seven pairs were striped and the rest were solid.

What percent of the pairs of socks were solid?

1. Ask students to solve the problems in #6.
2. Students select additional problems from the percent problem-solving worksheet. (#1-19) (Note: Problems on this sheet are organized by level of difficulty.)
3. Students complete the three problems: **Tim’s Tires, Raffle Ticket Sales, and** **Library Books**. (See Summative Assessment section.)
4. Extended Learning/Practice (Homework): Additional problems from the Percent Problems worksheet can be assigned.

**Formative assessment: None**

**Summative Assessment:** *Tim’s Tires, Raffle Ticket Sales, and Library Books*

**Preview outcomes for the next lesson:**

Students will use their knowledge of ratios, rates, unit rates, and percents to complete the CEPA.

**Lesson 14: Summative Assessment**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Class:\_\_\_\_\_\_\_\_\_\_\_

**1. TIM’S TIRES**

Tim needs to buy 4 new tires for his car.

He found the following three ads in his local newspaper:

|  |  |  |
| --- | --- | --- |
| **Steve's Tires** | **Tire Mart** | **Collins Tires** |
| Each tire: $63.55  Buy 3 and get one free! | Each tire: $46.25 | Each tire: $56.75  25% off the total purchase! |

1. Where would Tim get the best deal?
2. If Tim only needed one tire, where would get the best deal?
3. If he were buying two tires, where would he get the best deal?

**2. RAFFLE TICKET SALES**

The students at Hunking School were selling raffle tickets to raise money for the tornado victims.

* The eighth grade had 300 tickets to sell.
* The seventh grade had 200 tickets to sell.
* The sixth grade had 100 tickets to sell.

1. At the end of the first week, each grade had sold 40% of their tickets. How many tickets had each grade sold?
2. With one day left in the fundraiser, the eighth grade had sold 225 tickets, the seventh grade had sold 150 tickets, and the sixth grade had sold 60 tickets. Which grade was had the fewest tickets left to sell?
3. Estimate the percentages sold for each grade. Which grade came closest to its goal of selling all of its tickets?
4. The teachers also decided to help out and were given raffle tickets to sell. They sold 32 tickets. This was 80% of the tickets the teachers were given. How many raffle tickets were the teachers given to sell?

**3. LIBRARY BOOKS**

Brianna and Devin went to the library and checked out some books. 30% of the books that Brianna checked out were fiction. 50% of the books that Devin checked out were fiction books. Brianna and Devin’s friend Shannon said that Devin checked out more fiction books than Brianna. Is Shannon correct? Use examples when you explain your thinking.

**Lesson 14: Summative Assessment ANSWER KEY**

**1. TIM’S TIRES**

Tim needs to buy 4 new tires for his car.

He found the following three ads in his local newspaper:

|  |  |  |
| --- | --- | --- |
|  | **Tire Mart** | **Collins Tires** |
| Each tire: $63.55  Buy 3 and get one free! | Each tire: $46.25 | Each tire: $56.75  25% off the total purchase! |

1. Where would Tim get the best deal? **Collin’s Tire (Steve’s: 190.65; Tire Mart: 185.00; Collin’s Tire 170.25)**
2. If Tim only needed one tire, where would get the best deal? **Steve’s**
3. If he were buying two tires, where would he get the best deal? **Collins Tire**

**2. RAFFLE TICKET SALES**

The students at Hunking School were selling raffle tickets to raise money for the tornado victims.

* The eighth grade had 300 tickets to sell.
* The seventh grade had 200 tickets to sell.
* The sixth grade had 100 tickets to sell.

1. At the end of the first week, each grade had sold 40% of their tickets. How many tickets had each grade sold?

**Gr. 8: 120 Gr. 7: 80 Gr. 6: 40**

1. With one day left in the fundraiser, the eighth grade had sold 225 tickets, the seventh grade had sold 150 tickets, and the sixth grade had sold 60 tickets. Which grade was had the fewest tickets left to sell? **Grade 6**
2. Estimate the percentages sold for each grade. Which grade came closest to its goal of selling all of its tickets?

**Gr. 7 and 8 both sold 75% of their tickets. Gr. 6 only sold 60%.**

1. The teachers also decided to help out and were given raffle tickets to sell. They sold 32 tickets. This was 80% of the tickets the teachers were given. How many raffle tickets were the teachers given to sell? **40 tickets**

**3. LIBRARY BOOKS**

Brianna and Devin went to the library and checked out some books. 30% of the books that Brianna checked out were fiction. 50% of the books that Devin checked out were fiction books. Brianna and Devin’s friend Shannon said that Devin checked out more fiction books than Brianna. Is Shannon correct? Use examples when you explain your thinking.

**Shannon is only correct if Brianna and Devin checked out the same number of books.**

**Lesson 14: Percent Problem Solving Worksheet**

1. Sidney spent 30% of her monthly salary on a new laptop last month. Then she spent 50% of her salary to pay her monthly bills. She put the remaining $1,200 into her savings account. What was her monthly salary?
2. Mr. Stevenson has 160 cows, goats, horses, and chickens on his farm. 19% of the animals are cows, 28% are goats, 13% are horses, and the rest are chickens. How many chickens are on the farm?
3. Larry had $60. He spent $24 on a beach towel and sunscreen. He spent the rest on a bathing suit. What percent of the $60 did he spend on the new bathing suit?
4. Thirty-five percent of the people who visit Pinewoods State Park take a hike on the Sunset Trail Loop . If 175 visitors hiked the trail last Saturday, how many visitors total did Pinewoods State Park have that day?
5. There were 440 boys and 360 girls at the championship baseball game. What percent of the students at the baseball game were girls?
6. Ms. Spring spent 45% of her savings on a new television set that cost $810. How much money did she have in her savings account before she bought the television set?
7. Shari and Justin are starting a coin collection. The chart below shows how many coins they have collected so far. Complete the charts, showing:

* How many coins they have of each kind
* The percentages for each type of coin

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **SHARI** | | **JUSTIN** | |
|  | **# of Coins** | **%** | **# of Coins** | **%** |
| Silver Dollars | 9 |  |  | 40% |
| Half dollars | 6 |  |  | 10% |
| Quarters | 5 |  |  | 50% |
| TOTAL |  |  | 30 |  |

1. Joelle and Kyle started a collection of sports trading cards. The chart below shows how many cards they have collected.

Complete their charts, showing:

* How many cards they have of each kind
* The percentages for each type of card

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **JOELLE** | | **KYLE** | |
|  | **# of Cards** | **%** | **# of Cards** | **%** |
| Football | 10 |  |  | 25% |
| Baseball | 30 |  |  | 10% |
| Basketball | 5 |  |  | 50% |
| Hockey | 15 |  |  | 15% |
| TOTAL |  |  | 40 |  |

Use the information in this chart to solve the next three problems:

|  |  |
| --- | --- |
| **School** | **Number of Students** |
| Valley Middle School | 450 |
| Woodbury Middle School | 600 |

1. A survey was done at both schools to find out how much television students watch.

The survey found that 300 Valley School students watch more than 2 hours of television every night, while only 270 Woodbury School students watch more than 2 hours of television every night.

* What percentage of Valley students watch more than 2 hours of television every night?
* What percentage of Woodbury students watch more than 2 hours of television every night?
* Do more Woodbury students or more Valley students watch more than 2 hours of TV each night?

1. Woodbury Middle School has 250 girls and Valley Middle School has 275 girls.

Which school has a greater percentage of girls? Explain your answer.

1. At each middle school, 75% of the students play on a sports team.

* How many Woodbury students play on a sports team?
* How many Valley students play on a sports team?
* A third middle school, Lancaster School, has 300 students on a sport team. This is 75% of the total students at Lancaster. How many total students are at Lancaster School?
* Create three other ratios equal to 75%.

1. The students at Whittier Middle School were raising money to help a local family who lost their home due to a fire.

* Eighth grade had a goal of $300.
* Seventh grade had a goal of $250.
* Sixth grade had a goal of $200.

At the end of the first week, each grade had raised 40% of their goal. How much money had each grade raised?

With one day left in the fundraiser, the eighth grade had raised $225, the seventh grade had raised $200, and the sixth grade had raised $150. Which grade was closest to their goal?

Suppose each grade was only $25 short of meeting its original goal. Calculate the percentages raised for each grade. Which grade came closest to its goal of raising all their money?

1. Technologies Are Us has a $900 computer on sale for 30% off. Discount Technology has the same computer priced $800 on sale for 20% off. Which store has the better computer deal? How do you know?
2. Anna has 85% of her books on bookshelves. The rest of her books have been borrowed by friends. If there are 76 books on the shelves, how many books are in Anna’s total book collection?
3. To raise money for charity, the sixth graders at Fisk School want to sell magazines. 26% of the magazine sales will be donated to charity. If the class plans to donate $400, what is the total dollar amount of magazines they will need to sell?
4. Amy’s paycheck for Tuesday is 80% of her paycheck for Wednesday. Her total pay for Tuesday and Wednesday is $144. How much did Amy earn on Wednesday?
5. Kelly’s monthly salary is 20% more than Lynn’s monthly salary. Kelly’s monthly salary is $4,800. What is Lynn’s monthly salary?
6. During the lacrosse season, Marissa scored 25% more goals than Patty. The two girls scored a total of 63 goals. How many more goals did Marissa score than Patty?

**Lesson 14: Percent Problem Solving Worksheet ANSWER KEY**

1. Sidney spent 30% of her monthly salary on a new laptop last month. Then she spent 50% of her salary to pay her monthly bills. She put the remaining $1,200 into her savings account. What was her monthly salary?

**$6,000**

1. Mr. Stevenson has 160 cows, goats, horses, and chickens on his farm. 19% of the animals are cows, 28% are goats, 13% are horses, and the rest are chickens. How many chickens are on the farm?

**64 chickens**

1. Larry had $60. He spent $24 on a beach towel and sunscreen. He spent the rest on a bathing suit. What percent of the $60 did he spend on the new bathing suit?

**60%**

1. Thirty-five percent of the people who visit Pinewoods State Park take a hike on the Sunset Trail Loop . If 175 visitors hiked the trail last Saturday, how many visitors total did Pinewoods State Park have that day?

**500 visitors**

1. There were 440 boys and 360 girls at the championship baseball game. What percent of the students at the baseball game were girls?

**45%**

1. Ms. Spring spent 45% of her savings on a new television set that cost $810. How much money did she have in her savings account before she bought the television set?

**$1,800**

**ANSWER KEY**

1. Shari and Justin are starting a coin collection. The chart below shows how many coins they have collected so far.

Complete the charts, showing:

* How many coins they have of each kind
* The percentages for each type of coin

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **SHARI** | | **JUSTIN** | |
|  | **# of Coins** | **%** | **# of Coins** | **%** |
| Silver Dollars | 9 | 45% | 12 | 40% |
| Half dollars | 6 | 30% | 3 | 10% |
| Quarters | 5 | 25% | 15 | 50% |
| TOTAL | 20 | 100% | 30 | 100% |

1. Joelle and Kyle started a collection of sports trading cards. The chart below shows how many cards they have collected.

Complete their charts, showing:

* How many cards they have of each kind
* The percentages for each type of card

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **JOELLE** | | **KYLE** | |
|  | **# of Cards** | **%** | **# of Cards** | **%** |
| Football | 10 | 16.6% | 10 | 25% |
| Baseball | 30 | 50% | 4 | 10% |
| Basketball | 5 | 8.3% | 20 | 50% |
| Hockey | 15 | 25% | 6 | 15% |
| TOTAL | 60 | 100% | 40 | 100% |

Use the information in this chart to solve the next three problems:

|  |  |
| --- | --- |
| **School** | **Number of Students** |
| Valley Middle School | 450 |
| Woodbury Middle School | 600 |

1. A survey was done at both schools to find out how much television students watch.

The survey found that 300 Valley School students watch more than 2 hours of television every night, while only 270 Woodbury School students watch more than 2 hours of television every night.

* What percentage of Valley students watch more than 2 hours of television every night? **66.6%**
* What percentage of Woodbury students watch more than 2 hours of television every night? **45%**
* Do more Woodbury students or more Valley students watch more than 2 hours of TV each night?

**Valley students**

1. Woodbury Middle School has 250 girls and Valley Middle School has 275 girls.

Which school has a greater percentage of girls? Explain your answer.

**Valley Middle School 45.8% to 41.6%**

1. At each middle school, 75% of the students play on a sports team.

* How many Woodbury students play on a sports team? **450 students**
* How many Valley students play on a sports team? **337 students**
* A third middle school, Lancaster School, has 300 students on a sport team. This is 75% of the total students at Lancaster. How many total students are at Lancaster School? **400 students**
* Create three other ratios equal to 75%. **3.4, 6/8, 12/16, etc.**

1. The students at Whittier Middle School were raising money to help a local family who lost their home due to a fire.

* Eighth grade had a goal of $300.
* Seventh grade had a goal of $250.
* Sixth grade had a goal of $200.

At the end of the first week, each grade had raised 40% of their goal. How much money had each grade raised?

**Grade 6: $80 Grade 7: $100 Grade 8: $120**

With one day left in the fundraiser, the eighth grade had raised $225, the seventh grade had raised $200, and the sixth grade had raised $150. Which grade was closest to their goal? **Grade 7**

Suppose each grade was only $25 short of meeting its original goal. Calculate the percentages raised for each grade. Which grade came closest to its goal of raising all their money? **Grade 8 came closest to the goal.**

**(Grade 6: 87.5%, Grade 7: 90%, Grade 8: 91.6%)**

1. Technologies Are Us has a $900 computer on sale for 30% off.

Discount Technology has the same computer priced $800 on sale for 20% off.

Which store has the better computer deal? How do you know?

**Discount Technology has the best deal. $630 (Technologies Are Us: $640)**

1. Anna has 85% of her books on bookshelves. The rest of her books have been borrowed by friends. If there are 76 books on the shelves, how many books are in Anna’s total book collection?

**89 books**

1. To raise money for charity, the sixth graders at Fisk School want to sell magazines. 26% of the magazine sales will be donated to charity. If the class plans to donate $400, what is the total dollar amount of magazines they will need to sell?

**$1538.46**

1. Amy’s paycheck for Tuesday is 80% of her paycheck for Wednesday. Her total pay for Tuesday and Wednesday is $144. How much did Amy earn on Wednesday?

**$80**

1. Kelly’s monthly salary is 20% more than Lynn’s monthly salary. Kelly’s monthly salary is $4,800. What is Lynn’s monthly salary?

**$4,000**

1. During the lacrosse season, Marissa scored 25% more goals than Patty. The two girls scored a total of 63 goals. How many more goals did Marissa score than Patty? **Marissa scored 7 more goals than Patty.**

## Curriculum Embedded Performance Assessment (CEPA)

Pizza Champions

**Overview**

Students create a pizza recipe and plan ingredients to make pizza for 12, 60, and 240 students. Students compute unit prices for the ingredients, prepare a budget, and calculate the cost to feed varying numbers of people. Using ratio/rate language, each student writes a proposal to persuade the cafeteria manager to use his/her recipe. Students may work in pairs or small groups to discuss and plan the tasks, but all written work should be completed individually in order to assess each student’s understanding of proportional reasoning (ratios, rates, and unit rates).

**List of Tasks**

CEPA Task 1: Cost to Make One Pizza

CEPA Task 2: Cost to Feed 12, 60, and 240 Students

CEPA Task 3: Proposal to the Cafeteria Manager

**Teacher Instructions for CEPA Tasks 1, 2, and 3:**

Read the instructions on the handouts aloud and verify that all students understand the directions. (\*Note: During the pilot phase, please note whether students used calculators or computers, and record any additional directions/instructions given by the teacher.)

Students may plan and discuss this part of the task with other students. However, they should complete the work independently without help from teachers or peers.

The work produced during each task will be incorporated into the final project. Evaluation of each task will occur when the final product is submitted. All work must be completed during class time. CEPA Tasks 1 and 2 will each take one class period. CEPA Task 3 will take 1-2 class periods. Students are not permitted to take this work home until the project is finished and scored.

**Standards Addressed**

**6.RP.2** Understand the concept of a unit rate ***a***/***b*** associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*

**6.RP.3**Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

**SMP.4** Model with mathematics.

**WHST.2** Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.

Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples.

Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts.

Use precise language and domain-specific vocabulary to inform about or explain the topic.

Establish and maintain a formal style and objective tone.

Provide a concluding statement or section that follows from and supports the information or explanation presented.

**WHST.4**Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

This CEPA also addresses the writing portion of guiding principle 5 of the 2011 MA Curriculum Framework for Mathematics.

# Guiding Principle 5: Literacy across the Content Areas

*An effective mathematics program builds upon and develops students’ literacy skills and knowledge.*

Reading, writing, and communication skills are necessary elements of learning and engaging in mathematics, as well as in other content areas. Supporting the development of students’ literacy skills will allow them to deepen their understanding of mathematics concepts and help them to determine the meanings of symbols, key terms, and mathematics phrases, as well as to develop reasoning skills that apply across the disciplines. In writing, teachers should consistently support students’ ability to reason and achieve deeper understanding of concepts, and to express their understanding in a focused, precise, and convincing manner. Mathematics classrooms should incorporate a variety of written assignments ranging from math journals to formal written proofs.

**CEPA Task I: Cost to Make One Pizza: Directions for Teachers**

**Materials Required**

Calculators or computers with spreadsheet software

Directions for students

Price of selected ingredients

**Description**

Students create a recipe for pizza and use a price sheet to determine the cost of ingredients for their pizzas.

Using a template, students create a table showing the cost to make one pizza. The first line of the template is partially completed as an example. Students use a spreadsheet or calculator to compute unit costs for the pizza ingredients.

|  |
| --- |
| **CEPA Task 1: Student Directions**  You have created an award-winning recipe for pizza. You must write a proposal to convince the cafeteria manager to use your recipe. The proposal will include the cost to feed 12, 60, and 240 students. To help persuade the manager, you must explain all of the mathematics used to create your budget.  This CEPA involves three tasks.   1. First, you will find the cost to make one pizza using your recipe. 2. Second, you will determine the cost to make pizza for 12, 60, and 240 students. 3. Last, you will write a proposal to the cafeteria manager.   Use the price sheet to determine the ingredients for your pizza.  You may use a spreadsheet or calculator to perform the computations.  Use the template below to make a table showing the cost to make one pizza using your recipe.  The first line is partially completed as an example.  **Important information**:  Your pizza must include dough, sauce, cheese, and three other ingredients.  1 pound (lb) = 16 ounces (oz)  One 16-inch pizza serves 4 students. |
| **Award Winning Pizza Recipe**  **(Recipe for One Pizza)**   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **Ingredient** | **Amount Needed** | **Package Size** | **Price** | **Unit Cost**  **(cost per ounce)** | **Total Cost for 1 Pizza** | | Dough | 20 oz. | 60 oz. | $4.20 |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |

Price Sheet

|  |  |  |  |
| --- | --- | --- | --- |
| **Ingredient** | **Amount Needed** | **Package Size** | **Price** |
| Dough | 20 oz. | 60 oz. | $4.20 |
| Pizza Sauce | 5 oz. | 15 oz. | $1.20 |
| Cheese (shredded) | 8 oz. | 16 oz. | $3.36 |
| Pepperoni | 4 oz. | 12 oz. | $5.52 |
| Meatballs | 8 oz. | 16 oz. | $3.84 |
| Sausage | 8 oz. | 1 lb. | $4.16 |
| Green Bell Peppers | 8 oz. | 1 lb. | $1.44 |
| Onions | 8 oz. | 2 lbs. | $2.56 |
| Mushrooms | 6 oz. | 24 oz. | $4.56 |
| Tomatoes | 3 oz. | 12 oz. | $2.88 |
| Black Olives | 6 oz. | 6 oz. | $1.86 |

**CEPA Task 2: Cost to Feed 12, 60, and 240 Students**: **Directions for Teachers**

**Materials Required**

Calculators or computers with spreadsheet software

Directions for students

Price of selected ingredients

**Description**

Students determine the cost to make pizza for 12, 60, and 240 students and make a table or series of tables to show the following.

* the number of students
* how many pizzas are needed
* how much of each ingredient is needed
* the price of each ingredient
* the unit cost for each ingredient
* the total cost for each ingredient
* the total cost (based on unit rates) to feed 12, 60, and 240 students

**CEPA Task 2: Student Directions**

Determine the cost to make pizza for 12, 60, and 240 students. Remember, one 16-inch pizza serves 4 students. Create a table to show the following.

* the number of students
* how many pizzas are needed
* how much of each ingredient is needed
* the price of each ingredient
* the unit cost for each ingredient
* the total cost for each ingredient
* the total cost (based on unit rates) to feed 12, 60, and 240 students

You may use a spreadsheet or calculator to perform the computations.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ingredient | Unit Cost for 1 pizza | Pizzas Needed  for 12 Students | Total Cost for 12 Students | Pizzas Needed for 60 Students | Total Cost for 60 Students | Pizzas Needed for 240 Students | Total Cost for 240 Students |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
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**CEPA Task 3: Proposal to the Cafeteria Manager: Directions for Teachers**

**Materials Required**

Directions for students

Computers with word processing software

**CEPA Task 3: Student Directions:**

Write a proposal to convince the cafeteria manager to add your pizza to the school lunch menu. Your proposal should include the following.

* the tables you created for Tasks 1 and 2
* an explanation of the information provided in each table
* an explanation of the mathematics you used to create the information in each table
* a convincing argument for choosing your recipe.

**Criteria for Evaluating Student Work on the CEPA Tasks**

Student work will be submitted electronically for scoring at the state-level. Teachers may use the scoring guide below or any other rubric that they choose for their classroom-based scoring.

|  |  |
| --- | --- |
| **Possible Points** | **Mathematical Work**  **CEPA Tasks 1 and 2** |
| **4** | **Response demonstrates thorough understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.**   * The table is clear and accurately labeled. * The unit costs and/or costs for multiple servings are accurate. * The amounts and total costs are accurate. |
| **3** | **Response demonstrates strong understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.**   * The table is mostly clear and accurately labeled. * Most unit costs and/or costs for multiple servings are accurate. * Minor errors do not interfere with overall accuracy. |
| **2** | **Response demonstrates some understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.**   * The table may be somewhat unclear. * Some unit costs and/or costs for multiple servings are accurate. * Without additional work, it is difficult to present a solid argument for persuading the cafeteria manager to adopt the recipe. |
| **1** | **Response demonstrates weak or limited understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.**   * The table is unclear or mostly incorrect. * Few or no unit costs and/or costs for multiple servings are accurate. * Without significant support and additional work, it is impossible to present a solid argument for persuading the cafeteria manager to adopt the recipe. |
| **0** | **Response is missing, unrelated to the assigned topic, illegible, not of sufficient length to score, or shows no understanding of the standards addressed.** |

|  |  |  |  |
| --- | --- | --- | --- |
| **Score** | | **Written Explanation (CEPA Task 3)** | |
| 4 | | **Proposal demonstrates strong ability to use precise mathematical language to write explanations that are focused, precise, and clear. Tables are detailed, well-organized, and informative. All mathematical computations are correct and clearly explained.**   * Writing is exceptionally focused and clear. * Use of mathematical language is precise and sophisticated. * Tables are detailed, informative, and well-organized. * All mathematical computations are correct. * The presentation is highly convincing and thorough. | |
| 3 | | **Proposal demonstrates good ability to use mathematical language to write explanations that are focused, precise, and clear. Tables are mostly well-organized and informative. Mathematical computations are mainly correct and clearly explained.**   * Writing is focused and clear. * Mathematical language is used correctly. * Tables contain mostly correct information and are easy to interpret. * Mathematical computations are mostly correct. * The presentation is convincing. | |
| 2 | | **Proposal demonstrates some ability to use precise mathematical language to write explanations that are focused, precise, and clear. Mathematical computations contain some errors and need revision. Some additional work is required to transform this proposal into an effective argument for a particular pizza recipe.**   * Writing sometimes lacks focus and clarity. * Use of mathematical language is inconsistent. * Tables may contain some inaccurate information or be difficult to interpret. * With some support, the presentation could be transformed into a convincing proposal. | |
| 1 | | **The proposal demonstrates limited ability to use precise mathematical language to write explanations that are focused, precise, and clear. The tables contain inaccurate information. Significant support and revision are required to transform this proposal into an effective argument for a particular pizza recipe.**   * Writing lacks focus and clarity. * Mathematical language is missing. * Tables contain inaccurate information or are difficult to interpret. * The student requires significant reteaching and support to develop a viable argument. | |
| 0 | **Response is unrelated to the assigned topic, illegible, not of sufficient length to score, or shows no understanding of the standards addressed.** | |

## List of Unit Resources

**Lesson 1:**

Websites

**Lesson 2**:

Pictures of objects in scattered arrangements (provided by teacher)

Websites

Lesson 2 *In-Class Work*

Lesson 2 *Homework*

**Lesson 3:**

Websites

Lesson 3 *Homework*

**Lesson 4:**

Lesson 4 *In-Class Work*

Lesson 4 *Homework*

Websites

**Lesson 5:**

Lesson 5 *In-Class Practice*

*Ratios Top-Down Web*

**Lesson 6:**

Lesson 6 *In-Class Work: Ratio Review*

Lesson 6 *Ratio Assessment*

**Lesson 7:**

Lesson 7 *Formative Assessment: Rates*

**Lesson 8: None**

**Lesson 9:**

*Sam’s Market Weekly Sale Flyer*

*Justin’s Pizza*

**Lesson 10:**

*Little Red Riding Hood’s Grocery Trip*

**Lesson 11:**

Website

Lesson 11 *Homework*

*Battle of Fractions, Decimals, and Percents*

*Grids for Battle of Fractions, Decimals, and Percents*

10 x 10 Grids

100 Chart

**Lesson 12:**

*Tape Diagrams*

*Double Line Diagrams*

**Lesson 13: None**

**Lesson 14**:

Lesson 14 *Percent Problem Solving Worksheet*

*Tim’s Tires, Raffle Ticket Sales, Library Books*

**CEPA**

Pizza Champions

1. Expectations for unit rates in this grade are limited to non-complex fractions. [↑](#footnote-ref-1)