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| **Connecting Proportions, Lines and Linear Equations** |
| Mathematics, Grade 8 |
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| This unit develops an understanding of ratios, proportionality, linear functions, and slopes. Students will relate the constant of proportionality to slope and learn to recognize equations in the form of *y = mx* and *y = mx + b as linear functions because their graphs are straight lines*.Students will also represent linear functions in a variety of ways, including tables, graphs and equations. The study of functions in general is not explored in this lesson.The unit reflects Critical Area 1 in the 2011 MA Curriculum Framework for Mathematics**.**  *These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.* |

Table of Contents

[Lesson 1: Relationships between Quantities 8](#_Toc365307209)

[Lesson 2: Representing Proportional Relationships 19](#_Toc365307210)

[Lesson 3: Ratios and Rates of Change Numerically 35](#_Toc365307217)

[Lesson4: Analyzing Rates of Change Visually and Numerically 42](#_Toc365307218)

[Assessing Student Understanding for the Different Representations of Linear Relationships 51](#_Toc365307221)

[Lesson #6 Title: Deriving the equations y= mx and y=mx+ b 55](#_Toc365307222)

[Lesson #7 Title: Determining an equation of a line given 2 known points 61](#_Toc365307223)

[Lesson 8 Title: *EXTRA PRACTICE* 63](#_Toc365307224)

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| **Stage 1 Desired Results** | | | |
| ESTABLISHED GOALS  *8 EE 5 -*  Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.  *8 EE 6* - Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b.*  *8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.*  *8.F.2* *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*  *8.F.3* *Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear.*  *SMP1* - Make sense of problems and persevere in solving them  SMP2 Reason abstractly and quantitatively  SMP3 – Construct viable arguments and critique the reasoning of others  *SMP4* - Model with mathematics  SMP 6- Attend to precision  *ELA - SL8.4:* Presents claims and findings, relative evidence, valid reasoning and well chosen details | ***Transfer*** | | |
| *Students will be able to independently use their learning to…*  T3. Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems. | | |
| ***Meaning*** | | |
| UNDERSTANDINGS  *Students will understand that…*  U1. Proportional relationships can be represented in a variety of ways and proportional reasoning can be applied to real world situations.  U2. When two quantities are related proportionally the numerical values of both quantities change by the same factor.  U3. Proportional relationships represent a constant rate of change between two quantities and when graphed the relationship is linear.  U4. The steepness of non-vertical lines can be measured and this measurement is called the slope and in real world situations represents the rate at which two quantities change in relation to each other.  U5. Similar triangles can be used to show that the slope of a non-vertical line is a constant rate of change.  U6. Any two distinct points on a non- vertical line can be used to find the slope of a line. | | ESSENTIAL QUESTIONS  *Students will keep considering…*  Q1. What does it mean to be proportional*?* What does it mean to not be in proportion?  Q2. What story does a graph tell?  Q3. How are unit rates and other rates of change similar and/or different?  Q4. How can proportional reasoning help us make sense of real world situations?  Q5. How can similar right triangles help us understand the slope of a line? |
| ***Acquisition*** | | |
| *Students will know…*  K1. The lines of the equation *y = mx* pass through the origin.  K2 Slope is a constant rate of change.  K3. In a proportional relationship, when the input changes by a factor, then the output will also change by the same factor.  K4. Academic vocabulary: proportion, unit rate, constant rate of change, slope, similar triangles, input, , , origin, y-intercept, x-intercept, *y = mx*, *y = mx+b, relation, function,*  K5. A relationship whose graph is a straight line is called a linear function. | *Students will be skilled at…*    S1. Comparing two different proportional relationships represented in two different ways.  S2. Identifying the similarities and differences between *y = mx* & *y = mx + b* and identify the rate of change*.*  S3. Evaluating the slope when represented in different ways and comparing proportional relationships in a variety of  contexts (tables, graphs, equations, word problems).  S4. Modeling real world applications graphically.  S5. Identifying proportional relationships in a real world situation.  S6. Calculating the slope through two distinct points on the line.  S8. Deriving the equations of y=mx and y=mx+b | |

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| **Assessment Evidence** |
| ***CURRICULUM EMBEDDED PERFORMANCE ASSESSMENT***  **Title: Summer Work - Comparing Jobs**  **Goal**: Your job is to research different summer work opportunities and determine which opportunities represent the most feasible jobs for you to meet your personal goal.  **Role:** Analysts    **Audience:** Your parents  **Situation:**   1. You have asked your parents if you can to go to the NASA Space Camp in Huntsville Arizona next year. Because the camp is expensive (initial costs for Space Camp start at $779 plus transportation to and from your home), your parents said you would need to earn and save the money pay for it. Since you don’t have much money, you want to get a summer job. In order to help you reach your goal, your parents have offered to match the amount of money you earn over the course of the summer. (This means they will give you the same amount of money you earned at the end of the summer). 2. In order to know how much money you need to save, you need to estimate your costs for Space Camp. You need to consider travel to and from home by plane, and taxi for ground transportation to and from the airport. Are meals included in the cost of the camp? If not, you will need to consider money for meals. 3. In order to find the best job, you are to select at least 3 jobs from a list of summer job opportunities in the *Summer Opportunities Flyer.* Your task is to analyze these job opportunities using mathematical reasoning of linear relationships. You must include multiple representations (equations, tables, and graphs) for each job situation. 4. You will need to decide which job will allow you to earn the most money and support your decision with evidence of mathematical reasoning of linear relationships.   **Product/Performance:** Construct an analysis of the three job opportunities containing the following information:  A) Show and explain the income potential of the three different job opportunities.  B) Show and explain which of the three (or more) job opportunities will pay the most over the summer. Be sure to justify your reasoning!  C) Which job will be the best job for you? Explain why this is the case using the mathematical evidence youcollected in this performance task as well as your own interests and goals.  D) Present the information to your parents using varied media to convince them the job you’ve chosen is the best one for you. |

**Learning Plan**

Lesson 1: *Relationships between Quantities*

* *Location Graphs*
* Card Sort
* Formative Assessment- *Graphical Stories*
* *Football (Soccer) Distance Time Graph Game*

Lesson 2: *Representing Proportional Relationships*

* *Math Snacks- Ratey the Cat-* rates and unit rates
* *What Story Does a Graph Tell Us?*
* Function- define
* Introduce the CEPA
* Explore linear and non-linear relations with tables and graphs
* *Rule of Four*
* Determining whether a relationship between quantities represents a proportional relationship, linear function, both or neither

Lesson 3: *Ratios and Rate of Change Numerically*

* Understanding linear relationships using tables
* Compare different linear functions
* Identifying unit rate of lines

Lesson 4: *Analyzing Rates of Change Visually and Numerically*

* Determine slopes of lines from graphs, equations, and tables
* Compare proportional relationships in different ways
* Investigating slope
* Using similar triangles to explain why slope is the same between any two points on a non-vertical line
* Derive the equations y=mx and y-mx+b
* George and Steven’s savings plans

Lesson 5: Formative Assessment- *Assessing for Understanding of Different Representations of Linear Relationships*

* Interpret and compare unit rates to determine a better buy

Lesson 6: Deriving the Equations y = mx and y = mx + b

* Determine slope of any non-vertical line using similar triangles
* George and Steven’s savings plans (cont’)
* *The Race* problem

Lesson 7: *Determining the Equation of a Line Given Two Known Points*

Lesson 8: Extra Practice

* Illuminations: *Rise-run Triangles* to determine slope of a line
* Determine if slope is positive or negative

CEPA

## Lesson 1: Relationships between Quantities

**Brief Overview:** In this lesson, students explore and interpret the meaning of line graphs through three learning experiences using interactive software. In the first activity, students identify locations of people at various times during the day. Through a distance time graph in the second activity, students analyze and match graphs to videos clips and/or verbal descriptions. In the last activity, students participate in a card-sort activity in which they examine and match graphs with stories, and justify their decisions with evidence. The lesson culminates with students discussing the essential question: What story does a graph tell us? As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Estimated Time:** 1 hour

**Resources for Lesson:**

Internet access

*Location Graphs*: <http://www.teachersdomain.org/resource/rttt12.math.locgraphs/>

Card Sort Activity: <http://opi.mt.gov/pdf/CCSSO/InterpTimeDistance.pdf>

Handouts:

Formative assessment

Homework

Football (Soccer) Distance Time graph Game[http://www.sycd.co.uk/dtg](http://www.sycd.co.uk/dtg/)**[/](http://www.sycd.co.uk/dtg/)**

**Content Area/Course:** Mathematics **Grade(s):** 8 **Time:** Two 60-minSessions

**Unit Title:** ***Proportions, Lines, and Equations***

**Lesson #1 Title:** ***Relationships between Quantities***

**Essential Question(s) to be addressed in this lesson**: ***What story does a graph tell?***

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**8.F.5** *Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.*

***SMP1*** - Make sense of problems and persevere in solving them

**SMP2**- Reason abstractly and quantitatively

***SMP4*** - Model with mathematics

*By the end of this lesson students will know and be able to:*

Identify a graph that reflects a specific scenario between two quantities and create a scenario to reflect a specific graph.

**Instructional Resources/Tools**

*Location Graphs* **:** [**http://www.teachersdomain.org/resource/rttt12.math.locgraphs/**](http://www.teachersdomain.org/resource/rttt12.math.locgraphs/)

Card Sort Activity: <http://opi.mt.gov/pdf/CCSSO/InterpTimeDistance.pdf>

Handouts:

* Formative assessment
* Homework
* Football (Soccer) Distance Time graph Game **<http://www.sycd.co.uk/dtg/>**

**Anticipated Student Preconceptions/Misconceptions**

Students may not know that graphs can and do connect to real-world situations.

Students may have difficulty interpreting a distance-time graph versus a literal interpretation of the picture (the shape of the graph tells the story rather than understanding the shape depicts the relationship between distance and time).

Be conscious of ELLs and the vocabulary that may arise such as independent and dependant variables

**Lesson Sequence and Description**

**Introduce the Unit:**  Today we are beginning a unit on Connecting Proportional Reasoning, Lines, and Linear Equations. Last year you learned about proportional relationships. Take a minute to reflect and jot down several understandings you have about variables, graphs, ratios, proportions, proportional relationships, and graphs of proportional relationships. Now Turn and Talk with a classmate and discuss your reflections. Be prepared to share one or two with the class.

Activate prior knowledgeusing questions such as: Why do we use graphs?

What types of graphs have you seen or used? How does a graph tell a story?

Big Idea: A graph that shows a relationship between quantities may tell a story.

Communicate to students: Today our focus will be on line graphs (not to be confused with graphs of lines) and the story each one tells. For our first activity, we will use interactive software to interpret line graphs.

**Teacher Notes: Please read before beginning to teach this lesson.**

It is important to preview the resources in this lesson and work through the activities prior to teaching this lesson.

The purpose of this lesson is to check for students’ understanding of graph interpretation prior to beginning the more robust work of using proportional reasoning to connect the slope of a line with a rate of change between two quantities.

Parts of this lesson require students to have access to computers either in the classroom or in a computer lab.

Note: This two-day lesson is designed to be used in stations with students rotating through the stations over two days.

The plan is for the teacher to conduct a whole group modeling of task 1 using a Think-Aloud process followed by students completing Task 1.

NOTE: Task 3 may take an additional class period depending on students’ prior knowledge of distance and time graphs and their understanding of how a graph tells a story. Please refer to the full MARS lesson unit if you feel you need to develop the concept further.

Walk through the sample graph/location activity in Task #1 and model the reasoning involved using a Think-Aloud process (more information on Think-Aloud can be found at: **<http://www.rbteach.com/rbteach2/flash/videoplayer/streamer/mta/Modeling%20Thinking%20Aloud%20.pdf>** )

**Task #1: Location Graphs**

Link to website: [*http://www.teachersdomain.org/resource/rttt12.math.locgraphs*](http://www.teachersdomain.org/resource/rttt12.math.locgraphs)

**Location Graphs:**

Use Sample graph/location to orient students to the activity.

Click on the “**i**” on the lines as practice before using Location Graphs 1 & 2.

Use Location 1 and Location 2 with the whole class to identify which location fits the graph.

Have students justify their choice to the class or a partner. Possibly have the class critique the student’s choice before you drag the location into the answer box. (Note: before having classmates critique a student’s choice and reasoning, it is important that classroom norms are in place for providing feedback in a respectful fashion).

Follow up questions: For whole class discussion

How did the relationship between the number of people and the time of day help you determine the location? (ex. more/fewer people at different times of day- very early morning, late at night, etc.)

Would you say the number of people at a location depended on the time of day or vise-versa? The concept of independent and dependent variables is important for this unit. Expand upon the idea of independent and dependent variables by discussing:

Input (increment/step value)- in this case the time of day is independent. The time of day is used to find the number of people at a location.

Output-in this case, the number of people is the dependent variable- the number of people depends on the time of day- the value you get from using the independent variable

Reflection questions for students:

What does the cost of a car depend on?

What does the area of a rectangle depend on?

What does the size of a scale model of an airplane depend on?

What kinds of changes did you see in the number of people? (increasing and decreasing- gradually, quickly, no changes, etc.)

In grade 8 students are introduced to the term ‘function”. Use the following statement or a similar one: So, in this situation we can say that the number of people at a location is a **function** of the time of day. You can point out that the graphs depict **functions** because they show a relationship between input and output. The output depends on the input. Also, there is exactly one output for each input. This does not have to be delved into further at this time.

For more information, you may refer to the Background Essay and Teaching Tips listed on the activity site listed above.

**Task #2** **Applet Activity Football (Soccer) Distance Time Graph Game** [**http://www.sycd.co.uk/dtg/**](http://www.sycd.co.uk/dtg/)

The applet shows a soccer match (real) and a player dribbling the ball toward a goal. A distance-time graph follows the player’s actions and a line representing his constant speed is created by the applet. The player moves at various speeds and students are asked to match the graphs created to the videos and/or to verbal descriptions.

**Task #3** **Card Sort Activity**

Introduce the card sort activity to your students. Students should work in groups of 2, 3 or 4 with each group receiving a set of description cards and a set of graph cards.

Please use the following link to a unit with card sort templates: [InterpTimeDistance.pdf (application/pdf Object)](http://opi.mt.gov/pdf/CCSSO/InterpTimeDistance.pdf)

(if the live link does not work, paste this link into the address bar in your browser <http://opi.mt.gov/pdf/CCSSO/InterpTimeDistance.pdf>)

(pages S-2 and S-3 for Card Set A (graphs) and page S-5 for Card Set B (description)

Directions for the activity can be found on page 5 of the document:

Give each group the Card Set A: Distance–Time Graphs and Card Set B: Interpretations together with a large sheet of paper, and a glue stick for making a poster.

**Directions for students:**

You are now going to continue to explore matching graphs with a story, but this time as a group. You will be given ten graph cards and ten story cards. In your group, take a graph and find a story that matches it. Alternatively, you may want to take a story and find a graph that matches it. Take turns at matching pairs of cards. Each time you do this, explain your thinking clearly and carefully. If you think there is no suitable card that matches, write one of your own. Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them. Students may visit other groups to see how they matched the cards.

Solutions for Matching Graphs to Descriptions:

Graph Description

A 5

B 10

C 4

D 2

E 6

F 3

G 1

H 8

I 7

J 9

Card Sort Summary:To wrap up the card sort activity, have students share their matches. Start with Graph A and choose a group to explain their match. Have other groups decide whether this match is correct by supporting or refuting how the description matches the graph of distance and time. Continue this process through graph J.

At the end of class, have students discuss the essential question: *What story does a graph tell?*

Formative Assessment: Use Activity 5.2.1: Graphical Stories with students in pairs. Have the pairs complete the worksheet and then have each pair share a story.

Homework: See Homework Handout.

**Lesson 1 5.2.1: Graphical Stories Formative Assessment**

Below the following graphs are three stories about walking from your locker to your class.

**Two** of the stories correspond to the graphs. Match the graphs and the stories. Write stories for the other two graphs. Draw a graph that matches the third story.

Four graphs of a line with representing the relationshihp between the "distance from your locker" and time.
Graph A is a linear function. 
Graph B is in three parts; the first is a linear function, the second is a constant function, and the third is a linear function with a steeper slope than the first. 
The first part of Graph C is nonlinear; it shows a line increasing at a decreasing rate until it peaks and then begins decreasing at an increasing rate (an upside-down U shape). It reaches the x-axis and then increases as a linear function. 
Graph D is also non-linear. It begins with a steep slope and then increases at a decreasing rate. 

1. I started to walk to class, but I realized I had forgotten my notebook, so I went back to my locker and then I went quickly at a constant rate to class.

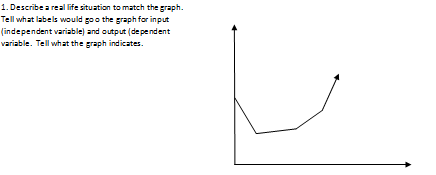
2. I was rushing to get to class when I realized I wasn’t really late, so I slowed down a bit.

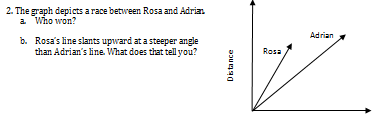
3. I started walking at a steady, slow, constant rate to my class, and then a teacher stopped me and asked me for a pass. Realizing I was going to be late, I ran the rest of the way at a steady, faster rate.

4. Write a story to describe the 4th graph (the graph not matched to the 3 situations above).

Lesson 1

Homework Lesson1





Time

*Sketch* a graph showing how the following quantities change in relation to one another:

1. Speed at which you walk; length of time it takes you to walk 5 miles
2. A person’s age; the person’s height

## Lesson 2: Representing Proportional Relationships

**Brief Overview:** In this lesson, students will continue to examine line graphs and determine the story they tell. They will begin to explore linear functions and their relationship to rate (constant rate of change). They will also determine whether or not a relationship is linear through tables, graphs, and equations. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:** In grade 7 students learned to plot points and graph lines, the difference between a ratio and a proportion, and how to identify the constant of proportionality in tables, graphs, and equations.

**Estimated Time:** 1 hour

**Resources for Lesson:**

Internet access

[**http://mathsnacks.com/ratey.html/**](http://mathsnacks.com/ratey.html/)

Handouts

* What Does a Graph Tell Us?
* Rule of Four
* Day 2 class work and homework

**Content Area/Course:** Mathematics **Grade(s):** 8 **Time:** Two sixty-min. sessions

**Unit Title:** ***Proportions, Lines and Equations***

**Lesson #2 Title:** ***Representing Proportional Relationships***

As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Essential Questions:**

*What does it mean to be proportional? What does it mean to not be in proportion?*

*What story does a graph tell?*

**Guiding Questions:** *How do graphs and/or tables depict proportional relationships?*

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

***8.EE.5***  Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

**8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[[1]](#footnote-1)

**8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

***8.F.3***Interpret the equation *y = mx + b* as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*

**SMP2**  Reason abstractly and quantitatively

**Assumptions about what students know and are able to do coming into this lesson:**

*In grade 7 students learned to plot points and graph lines, the difference between a ratio and a proportion, and how to identify the constant of proportionality in tables, graphs, and equations.*

**Instructional Resources/Tools :** [**http://mathsnacks.com/ratey.html/**](http://mathsnacks.com/ratey.html/), class work handouts

**Anticipated Student Preconceptions/Misconceptions**

Students may not remember how to determine which variable is dependent and which is independent in relationships, thus may not be able to express it correctly in the graph.

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| **Lesson 2 Sequence and Description**  ***DAY I Proportional relationships***   1. **Launch**: Watch the 3 minute video *Ratey the Cat* on *Math Snacks* (<http://mathsnacks.com/ratey.html> ) then discuss the concepts of rates and unit rates. Have students complete the Learner Guides 1 & 2 if you feel the class needs to review unit rate concepts. 2. **Lesson 2 Handout 8.5.1: What Does a Graph Tell Us? (p. 15 of this unit)**   **Teacher Notes for “What does a Graph Tell Us?**  **Table:** From the table you can see proportional relationships between the ordered pairs. **The ratio of earnings/hours is a constant.** To move from the table to the graph students will have to determine the labels for the axes. Generally when there is a rate involving time as in per hours, the x-axis is labeled with the hours (the “per” unit) and is the independent variable.  **Graph:** The graph shows a linear relationship between hours and earnings. (a constant rate of change). Have students state the change as *For every unit increase in time her earnings increase by $6.00 as seen by the arrows*.  **Function/Equation:**  Be sure students convey that there is a constant by which hours are multiplied (unit rate). A function is a relationship between quantities. When the graph of an equation (also a relationship between two quantities) is a straight line, that relationship is called a linear function.   1. **Lesson Use a Think-Pair-Share protocol (15 minutes)** 2. Have students individually work on answering the questions in problem 1 about Dakota (*What Does a Graph Tell Us?*). 3. Have them share their work with a partner and compare their work. 4. Once student pairs had enough time to work on Problem 1 begin asking questions to get at their reasoning about the relationship in the table and graph:  * What did Dakota’s earnings depend on? * How does a table show a relationship between the quantities, hours and earnings. (Have students write some proportions involving any two ordered pairs in the table) * What kind of relationship is it? * Does the graph show the same relationship? Why or why not? * Write a function relating the hours worked to the money earned. * How does this function describe the relationship between hours and earnings? * How does the graph illustrate proportionality?   **Function**: *Let students know we use the word function to indicate that in a relation between two quantities/variables* ***one variable depends on the value of another variable****: The value of y (earnings) in the equation* **y = 5x** *depends on the value of x (hours).*  *To further understand the input/output function concept* have students consider the following idea: If substituting (inputting) the number 3 for x in a formula could give you (output) 5 one time, but (output) 2 another time, what problems could that pose?  Let students also know that a function that is graphed as a ***straight line*** like the one in the problem is called *a* ***linear function****.*  Furthermore, this problem about Dakota’s earnings reveals that ***linear functions involve proportions-*** equal changes in one quantity are matched by equal changes in another quantity.  Ask how does the equation y = 6x connects to the idea of equivalent ratios for each ordered pair (Ask students if the equation y/x = 6 is equivalent to the equation y = 6x). Ask them to compare the values in the tables and where they see equivalent ratios there. The equation y = 6x can be interpreted directly in terms of ratios: Each y - value is 6 times as great as its corresponding x–value.  **Connect the idea that a ratio is a multiplicative comparison of 2 quantities (how many times greater is each y- value compared to each x value) or it is a joining of two quantities into a single unit (y/x). Ordered pairs (x, y) form the ratio y/x.**  Consider the ordered pairs (1,6), (2,12), (3,18), etc. Ask: *How are equivalent ratios seen in the ordered pairs?*  Explore: Ask students to examine the function y = ½ x (y equals one half of x) by creating a table of values over the integers. Ask: *Do equal changes in x result in equal changes in y?* *What do you think the graph would look like? Is it a linear function?*  Introduce the *Curriculum Embedded Performance Assessment* (CEPA) using an overhead projector. Explain to the students that they are not expected to know how to complete the CEPA at this point, but that they will be learning about the concepts throughout the unit that will prepare them for the CEPA. Note: *The goal here is not to have students begin to work on the CEPA, but rather for them to know what they will be expected to do by the end of the unit.*  Homework: Have students complete the *Rule of Four Link Sheet* for homework. Be sure to check in on responses to the question on variable dependency. This will help you assess whether you need to review dependent/independent variables with students.  **DAY 2**  Explore linear and non-linear relations with tables and graphs  Note: A proportional relationship is a linear function that passes through the origin and can be represented in multiple ways: verbally (spoken or written, numerically (tables), visually (graphs) and symbolically (equations).   1. Review the homework problem with students. The problem should reaffirm their understanding that a graph of a line represents a linear function and that linear functions involve proportional relationships meaning equal changes in one quantity (such as the hours increasing by 1 each time) are matched by equal changes in another quantity (such as the distance increases by seven each time). This relationship can be represented in multiple ways: graphically, numerically (table of values), and symbolically (by an equation) as well as verbally in the statement of the problem itself.   Linear functions involve a constant rate of change. Ask students where they see this constant rate of change in the graph, table, and equation.   1. **Paratrooper Problem** (*Lesson 2 Day 2 Handout* )   The intent of this activity is for students to recognize the relationship between the quantities is not linear, i.e., **changes in one quantity are not matched by equal changes in the other quantity.** It is important for students to understand **not all relationships are linear**. We use tables, graphs and equations to help us determine when relationships between two quantities are linear or not linear.   * There is a pattern in the table but the change in *distance* fallen is not the same each time for each change in *time* falling. * The graph is not a straight line. * **1.** Ask students to think about whether a paratrooper’s fall when jumping from an airplane would result in a linear relationship (function) between *time falling* and *distance fallen*. Have pairs of students *Turn and Talk* to discuss their thoughts with a partner. * **2**. Problem I -Handout the paratrooper problem sheet to students. Have them complete the questions and compare their responses with a partner. Ask them to be prepared to share something they said in common. Have each pair of students share something different each time. (See notes from Day 1 for Graph, table, and Equation).Continue to use the term *function* when opportunities arise   In the whole group share out encourage the groups to share something different. Acknowledge all contributions.  For the second problem, students use tools (tables and graphs) to justify their conclusions.   * **3.** Pose the question to students: *What do you think about the relationship between the radius of a circle and the circumference of the circle?* Have students turn and talk to discuss this question. (Follow the same protocol as in Problem I above).   You may need to help students recall the formula which they will use to create a table and then graph the values in the table. Discuss what is independent and dependent in the relationship.  After the whole class share out, discuss the formula for circumference in terms of how it compares to equations of the form y = kx (proportional equations). *What is the multiplier? What is the unit rate?*  Write some proportions using any two ordered pairs (Example: (2, 2, ( 3, 6 ) Write the formula as **C/r = 2**  4. On the same handout have students complete the *Check for Understanding* questions.  Homework*: Lesson 2 Day 2 Homework* handout. Students may create a graph or use the tables to determine if proportional relationships and/or linear functions exist.  **Teacher Reflection**  *What went well in this lesson?*  Can students determine which relationships are linear functions by a graph or a table?  *Did all students accomplish the outcome(s)?*  What evidence do I have? What would I do differently next time? |

Lesson 2 handout 8.5.1: What Does a Graph Tell Us?

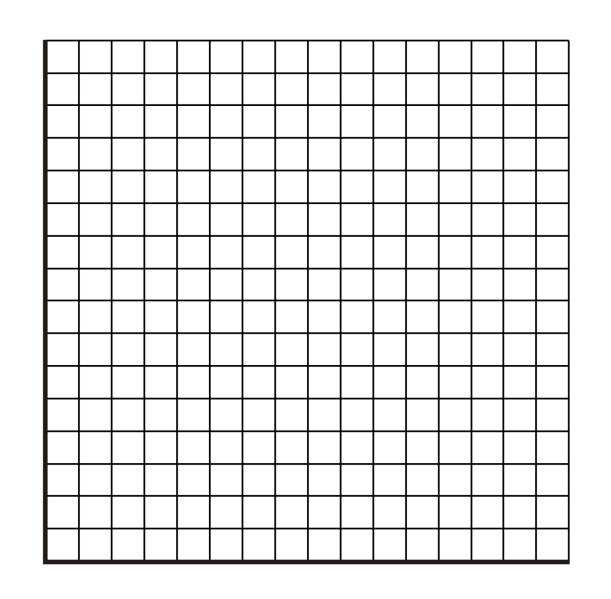
1. Dakota earns $12.00 helping her cousin deliver pizzas for 2 hours.

1. What would be Dakota’s hourly rate of pay for helping her cousin? Be precise about using units.

|  |
| --- |
|  |

1. If she worked 5 hours instead of 2 hours, what would she earn helping for 5 hours?

|  |
| --- |
|  |

1. Complete the table below for the hours she worked.

|  |  |  |
| --- | --- | --- |
| Hours Worked | Dollars Earned | $ Per Hour |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
|  |  |  |

Graph this relationship.

What patterns do you see in the table?

|  |
| --- |
|  |

Graph the relationship between hours and earnings. (Determine first which of the quantities is dependent on the other quantity). Let the x – axis be the independent variable.

1. Describe the graph. What patterns do you see in the graph?

|  |
| --- |
|  |

1. Write an equation that represents the relationship between her hours worked and her earnings.

|  |
| --- |
|  |

1. How much would she earn if she worked for: 8 hours? 20 hours? EXPLAIN! Did you use the table, graph, or equation?

|  |
| --- |
|  |

**THINK-PAIR-SHARE**

By yourself think about each question below and jot down some thoughts. Then turn and talk to a partner and discuss questions. Jot down some more thoughts. Be prepared to share your discussions with the class.

|  |
| --- |
| 1. Only non-negative numbers were used with this problem. Could negative numbers be used for this function? Why or why not? |
| 2. Could you use the graph to find the amount Dakota would earn for a given number of hours? Explain your reasoning. |
| 3. Could you use the graph to find out how long it would take Dakota to earn a certain amount of money? Explain your reasoning. |
| 4. If Dakota got a raise, how would the graph be similar to this graph? How would it be different? Explain your reasoning. |

**Rule of Four Handout**

|  |  |
| --- | --- |
| **Problem Solving Approaches to show our reasoning** | |
| **Verbal Description** | **Table** |
| **Jordan starts at his home and bikes along a country road at an average speed of 7 mi/hr.**   1. Show this relationship relating the length of time he bikes to the distance he travels in the, table, graph and as an equation. Use Labels appropriately. 2. Which representation will you use to find: 3. the number miles he biked in 12.5 hours? Why? 4. the number of hours it took him to bike 63 miles? Why? 5. Which can you use to find the rate of change in the relationship? Why? | Communicate Numerically   |  |  | | --- | --- | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |
| **Graph** | **Equation(s)** |
| Communicate Visually    a first quadrant coordinate grid | Communicate Symbolically (algebraically) |

Lesson 2 handout Teacher’s Notes 8.5.1: What Does a Graph Tell Us?

1. Dakota earns $12.00 helping her cousin deliver pizzas for 2 hours.

1. What would be Dakota’s hourly rate of pay for helping her cousin? Be precise about using units.

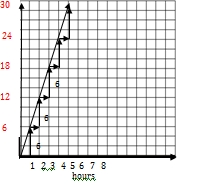
|  |
| --- |
| Rates, ratios, and Unit Rates, units $ 6 / hr |

1. If she worked 5 hours instead of 2 hours, what would she earn helping for 5 hours?

|  |
| --- |
| Proportional relationships $30 |

1. Complete the table below for the hours she worked.

|  |  |  |
| --- | --- | --- |
| Hours Worked | Dollars Earned | $ Per Hour |
| 1 | 6 | 6 |
| 2 | 12 | 6 |
| 3 | 18 | 6 |
| 4 | 24 | 6 |
| 5 | 30 | 6 |
| 6 | 36 | 6 |
|  |  |  |

Graph this relationship. 

What patterns do you see in the table?

|  |
| --- |
| Equivalent ratios 6/1, 12/2, 18/3 etc  Multiply the hours by 6 to find the total earnings  etc  The Table shows a **constant rate of change $/hr = 6** |

Graph the relationship between hours and earnings. (Determine first which of the quantities is dependent on the other quantity). Let the x – axis be the independent variable. The amount she earns depends on the hours she works. The number of hours she works determines how much she will make.

1. Describe the graph. What patterns do you see in the graph?

|  |
| --- |
| The graph can also be a set of points that form a line. Students will see each point is 6 units above and one unit right of the point before it. The graph depicts a linear function. Students can see the unit rate in the graph as the vertical increase, in the table as earnings per hour, in the equation as the multiplier of x. |

1. Write an equation that represents the relationship between her hours worked and her earnings.

|  |
| --- |
| Y = 6x, y/x = 6, |

1. How much would she earn if she worked for: 8 hours? 20 hours? EXPLAIN! Did you use the table, graph, or equation?

|  |
| --- |
| Students can extend the table, use the equation or add to the table to find she earns $48 working 8 hours. The equation would be easiest to find her earnings for working 20 hours, $120. |

**THINK-PAIR-SHARE:** **Teacher Notes**

By yourself think about each question below and jot down some thoughts. Then turn and talk to a partner and discuss the questions. Jot down some more thoughts. Be prepared to share your discussions with the class.

|  |
| --- |
| 1. Only non-negative numbers were used with this problem. Could negative numbers be used for this function? Why or why not?  Making sense of units |
| 2. Could you use the graph to find the amount Dakota would earn for a given number of hours?  Ordered Pairs (x, y) for (hours, earnings)  If you extend the graph, find what the y value is for the desired number of hours worked. |
| 3. Could you use the graph to find out how long it would take Dakota to earn a certain amount of money?  Find the x value that pairs with the amount she earned. |
| 4. If Dakota got a raise, how would the graph be similar to this graph? How would it be different?  The unit rate would be greater so the line would be steeper. |

### Lesson 2 Day 2 Handouts

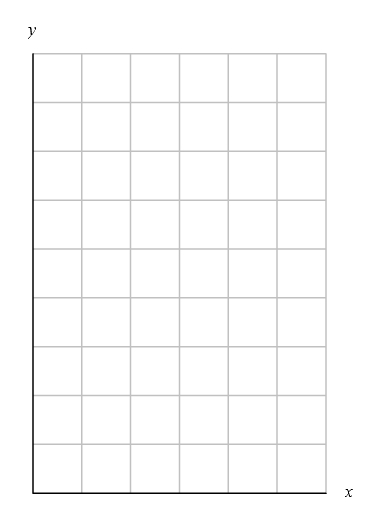
1. A paratrooper jumps from a plane. Is the relationship between the time she takes to fall and the distance she falls a linear function?

The data from the paratrooper’s fall is recorded in the table below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time Falling (seconds) | 1 | 2 | 3 | 4 | 5 |
| Distance Fallen (feet) | 12 | 48 | 108 | 192 | 300 |

What do you see happening in the table that helps you determine whether the relationship is Linear or non-linear?

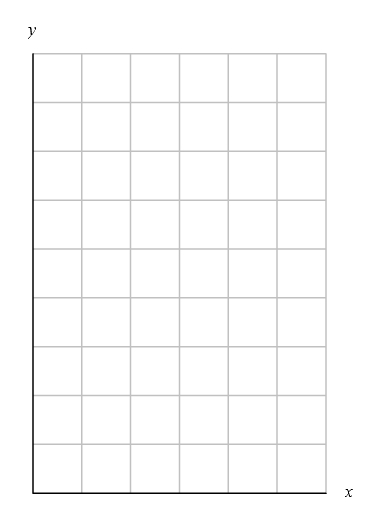
Check your thinking by making a graph of the data. Create scales for the axes and label them.



What do you notice about the graph?

Look for patterns to see if you can you find an equation that will fit the data.

1. Investigate the relationship between a circles’ **radius** and its **circumference** to determine if it is a linear relation or not. Use the tools below to help you answer the question. Use the graph and the table to justify your conclusion.



My justification:

|  |  |
| --- | --- |
| Radius | Circumference |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

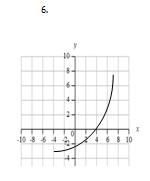
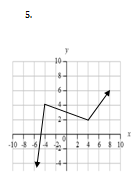
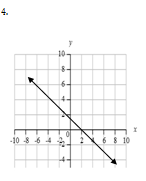
Check your understanding

|  |
| --- |
| How can you tell whether a relationship between quantities is linear from a table of values? |
| How can you tell whether a relationship between quantities is linear from a graph? |
| For non-linear functions is it possible to write a true proportion with any two ordered pairs? |

**Lesson 2 Day 2 Homework**

In questions 1-6, tell whether the relationship between the quantities represents a proportional relationship, a linear function, both, or neither. Explain your reasoning.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time swimming (min) | 1 | 2 | 3 | 4 | 5 |
| Number of arm strokes | 13 | 26 | 52 | 80 | 106 |
| 1. | | | | | |
| Cost of Beverage ($) | 1.00 | 3.00 | 5.00 | 6.00 | 7.00 |
| Sales Tax ($) | .07 | .21 | .35 | .42 | .49 |
| 2. | | | | | |
| Radius of circle (in) | 1 | 3 | 5 | 7 | 10 |
| Area of circle (in 2) | π | 9π | 25π | 49π | 100π |
| 3. | | | | | |



Write an equation relating the following quantities. Then graph the equation.

|  |
| --- |
| 7. The number of sneakers at $35.00 a pair; the cost of pairs purchased. Label your axes.  a coordinate gird for studetns to graph the problem.  Is the relationship linear? Proportional? Neither? Explain. |

# Lesson 3: Ratios and Rates of Change Numerically

**Brief Overview:** In this lesson, students will graph proportional relationships, interpreting the unit rate as the slope of the graph. They will compare two different proportional relationships represented in different ways.They will work to determine unit rates and the slope from graphs, equations, and tables. Finally, they will begin to use similar triangles to explain why the slope is the same between any two points on a non-vertical line in the coordinate plane*.* As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Estimated Time:** 1 hour

**Prior knowledge required**

**Resources:** Homework handout

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Content Area/Course**: Mathematic**s** **Grade(s):** 8  **Time:** 60mins  **Unit Title:** *Proportions, Lines, and Equations*  **Lesson #3 Title:** *Ratios and Rates of Change Numerically*  **Essential Question(s) to be addressed in this lesson*:***  *How are unit rates and other rates of change similar and/or different?*  **Standard(s)/Unit Goal(s) to be addressed in this lesson:**  **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.  **8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*.  **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.  ***SMP1*** - Make sense of problems and persevere in solving them  **SMP2**  Reason abstractly and quantitatively  ***SMP4*** - Model with mathematics  **Outcome(s)**  By the end of this lesson students will know and be able to:  *Determine unit rates and the slope from graphs, equations, and tables.*  **Lesson 3 Sequence:**   1. Homework review 2. Students work in groups of 3-4 to review their homework and discuss the questions posed in the problem. 3. Share out different groups responses.   Three different sets of tables (shown below) develop students’ understanding of linear relationships and the similarities and differences between their ratios/rates, equations and graphs. The first set of tables (Tables 1, 2, & 3) show three lines with different rates of change when graphed (Table 1 is proportional: y = ½ x; Table 2 is linear; Table 3 is linear). The objective is that students will move between tables, equations, and graphs while developing their understanding of rates of change of lines and graphs of linear function**s.**  For tables 2 and 3, the ratio of y/x is not constant, but the graphs show linear functions which have a constant rate of change. In Table 2, as x goes from 0 to 1 (increasing by 1 unit), y goes from -2 to -1 (increases by 1 unit). As x goes from 1 to 2 (increasing by 1 unit), y goes from -1 to 0 (increases by 1 unit). The rate (ratio) of change is 1 to 1. In Table 3, for each increase in x, y decreases by 2. The rate of change is negative.  Note: *All proportional relationships are linear functions- y=mx. Not all linear functions are proportional relationships, but all linear functions involve proportional relations- y=mx+b. This equation represents a vertical shift (of b units) of the proportional relationship y=mx.*  **Part A**  **Table 1**   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 1 | 2 | 3 | | Y: | 0 | 1/2 | 1 | 3/2 |   **Table 2**   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 1 | 2 | 3 | | Y: | -2 | -1 | 0 | 1 |   **Table 3**   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 1 | 2 | 3 | | Y: | 2 | 0 | -2 | -3 |  1. For each of Tables 1-3, ask students to determine whether or not the ratios of y-values to x- values are the same for each pair of x and y coordinates. *Which tables show ratios that remain constant, in other words, were all ratios equivalent?* 2. Have the students graph the sets of points from the three tables. Ask students how the graphs are similar and different (directions, steepness, crossing the axes, etc.). Check in with students about vocabulary terms they are familiar with: y-intercept, origin, slope, etc. 3. Ask students to determine a *rate of change* for each table*.*  You may want to ask it this way: *For each* ***unit*** *change in the x-value how does the y-value change?* (increases (or decreases) by ­­­\_\_\_ units*). Is this change the same each time (constant)? What is the “change in x” each time? What is the “change in y” each time?* 4. For each graph ask students to highlight 2 points on the graph. Ask them to find the ratio of the *change in y* to the *change in x* between those two points. For each table have students select a different pair of points and find again the ratio of the “*change in y”* to the “*change in x”*.   **Part B**  Note: In grade 7, students learned that in a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change. An infinite set of equivalent ratios is a rate.  Have pairs of students analyze and compare Tables 4 & 5 below. *Turn and Talk* to discuss their observations. Select a few pairs of students to share with the class.  Table 4   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 2 | 4 | 6 | | Y: | 0 | 6 | 12 | 18 |   Table 5   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 2 | 4 | 6 | | Y: | 2 | 8 | 14 | *20* |   *Have s*tudents graph the two tables of values in Tables 4 & 5. Ask them what they notice about the graphs. *How are the graphs for Tables 4 & 5 different? How are they the same? What do the similarities and differences in the equations indicate?*  Students will likely talk about the steepness of the lines (teacher should reference the steepness as *slope* and determine who is familiar with the term), and whether or not the lines pass through the origin or cross the axes at another point. They may also note the lines are parallel. Students should be guided toward this observation and language if they do not do so on their own.  Students should recognize that Table 4 shows a proportional relationship, whereas Table 5 does not. The ratio of y/x is the same for all variable pairs in Table 4 but not in Table 5. However, the ratios of the rate of changes in y to corresponding changes in x are equivalent.  Students can recognize the unit rate is the same for both lines. Ask what that means for the graphs and for the equations. By graphing and comparing the graphs of the two lines, students may be able to determine the equation of the Table 5 line by noticing each point is 2 units above the table 4 line and therefore each y-value in the ordered pairs is 2 more for an equation y = 3x + 2.  You may want to ask if the graph of Table 5 could reflect a proportional relationship between its variables. How?  After students have reported out, introduce the language of *y-intercept (where the line intercepts the y-axis)* if students are unfamiliar with the terms.  Have students determine the unit rates. Be precise in saying that *for each unit increase in the x-value the y-value increases (or decreases) by -------------units.*  Ask students to write the equations of each line. Table 4 is a proportional relationship, so it should be easy: y = 3x. Students can use the graphs of the lines to notice that the points on line 5 and therefore line 5 is a vertical shift of line 4 by 2 units. So the equation for line 5 would be y = 3x + 2. For any given x-value, the corresponding y-value for line 5 is 2 more than the y-value for line 4.  Have pairs of students analyze and compare Tables 6 & 7 below.. *Turn and Talk* to discuss their observations. Select a few pairs of students to share with the class.  Note: See *Lesson 3 Teacher Notes* (page 31) for further explanation for Tables 4 and 5 above as well as Tables 6 and 7 below.  **Part C**  Table 6   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 3 | 6 | 9 | | Y: | 0 | -5 | -10 | -15 |   Table 7   |  |  |  |  |  | | --- | --- | --- | --- | --- | | X: | 0 | 3 | 6 | 9 | | Y: | 5 | 0 | -5 | -10 |   Have students graph the two tables of values in Tables 6 & 7. Ask them what they notice. *How are the graphs for Tables 6 & 7 different? How are they the same? Can they determine the rate of change for each graph?*  In Table 6, the relationship is proportional: -5/3. Ask what that means in terms of the unit rate. (For each unit increase in x, y DECREASES by five thirds units). Have students write the equations for each table. For table 7, they can compare the graphs and note a vertical shift of +5.  Note: Students will learn about the concept of negative rate in a subsequent lesson. For now, use the phrase “increases by” or “decreases by”.  **Closure**  **Reflection Questions**   1. How are proportional and linear relationships alike/ different?  * We want students to conclude that they both have constant rates of change. The proportional relationship each ordered pair has the same ratio. (x, y ) , y/x. Their graphs are lines.  1. How could you determine a linear relationship by looking at a table, a graph, an equation?    * The equations look like y=kx where k is constant multiplier or y = kx + c where c is a vertical movement    * The graph looks like a straight line where for each point can be determined from an initial point by a constant change vertically and a constant change horizontally.    * The table shows a constant change in one quantity and a constant change in the second quantity.   **Homework / Formative Assessment**  Peter purchases his daily school lunch at the following rate:   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Number of Days | 0 |  | 5 |  | | Cost | 0 |  | 11.25 |  |   Paul purchases his school lunch as described by the equation: where x represents the number of days.  Who is purchasing lunches at a more economical rate, Peter or Paul? Explain your reasoning in writing using evidence from the problem. |

# Lesson4: Analyzing Rates of Change Visually and Numerically

**Brief Overview:** This lesson focuses on the graphing proportional relationships and interpreting the unit rate as the slope. They will *determine slopes of lines from graphs, equations, and tables. Students will continue to examine slope through similar triangles. Lastly, they will begin to explore the equations y = mx and y = mx + b*. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Estimated Time:** 1 hour

**Resources- Bob and Jeff handout**

**Content Area/Course**: Mathematic**s** **Grade:** 8  **Time:** 60 mins

**Unit Title:** Proportions, Lines, and Equations

**Lesson #4 Title**: *Analyzing Rates of Change Visually and Numerically*

**Essential Question(s) to be addressed in this lesson*:***

*How can proportional reasoning help us make sense of real world situations?*

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

**8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*.

***SMP1* -** Make sense of problems and persevere in solving them

**SMP2** Reason abstractly and quantitatively

***SMP4*** - Model with mathematics

**SMP6**- Attend to precision

By the end of this lesson students will know and be able to:

*Determine slopes of lines from graphs, equations, and tables.*

**Instructional Resources/Tools**

## Lesson 4 Handout- *Bob and Jeff*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Lesson Sequence and Description**   1. Review the homework problem of Peter and Paul.   Students work in pairs- Using a *Turn and Talk* protocol, have students share their explanation to the following question from the homework:  *Is Peter or Paul purchasing lunches at a more economical rate? Explain your reasoning using evidence from the problem.*  Have select pairs of students report out their explanations. Look for explanations that show comparisons of rate of change either by unit rates or graphs. This activity compares steepness of lines to rates of change. In a real world context we use the term rate of change to describe the relationship between the two quantities, e.g., the cost of lunches per number of lunches purchased. On a graph the same rate of change is depicted by the steepness of the line and is called the *slope* of the line. The slope of a line is the rate of change between the variables. At this point in the lesson, students should make a connection between the steeper the line, the bigger the unit rate.   * Write the equations for both functions and show the graphs of each function on the board. * Write the rates of change for each as ratios; decimals. * Compare the steepness of the lines to the values of the rates of change.   Have students consider a third student whose daily lunches are $3.00. Where would the graph of the line representing his total lunch costs over time be graphed in comparison to Peter and Paul’s?   1. ***Bob and Jeff*** Handout (page 32) *Think, Pair, Share*   Ask students to individually work on the problem for a few minutes, pair up with another student, and discuss the problem and their conclusions. Students should be ready to share with the class.  Include in the share out the following questions:    How much time did it take?  What is Bob’s unit rate?  What is the slope of the line in Bob’s graph? (hint: what is the unit rate?)  What is the meaning of the origin in Bob’s graph?  What is the equation of Bob’s graph?  What is Jeff’s unit rate?  What is the equation of Jeff’s walking rate with respect to distance and time?  What would be the slope of the line represents Jeff’s walking rate?  Do the graphs of Bob and Jeff’s walking rates represent a proportional relationship? How do you know?   1. **Investigating slope (steepness)**   **Note: Use an overhead graphing calculator or white board applications for this part of the lesson.**  **Part A: Slope of y = mx**  Have students work in groups of 3-4 to use a graphing utility ( or graph paper) to graph the lines  y = ax for several different values of *a*. Be sure to include fractions and negative numbers for *a*. Note: The time units may cause students to make errors graphing Jeff’s rate. His rate is 4 miles per hour.   * Have students discuss the similarities and differences they observe. * What conclusions can they make about the steepness and the constant “a”? * Use the word *slope* and discuss real world connections the word I has to steepness.   Have students note the steepness of the lines. How does the value of *a* affect the steepness?  Introduce the word slope as the numeric value that is used to describe the rate of change for a linear function.  Note: A rate is a set of infinitely many equivalent ratios. The line itself represents the slope because it is a proportional relationship.  **Part B: How is the slope determined from a graph:**  **How could you maintain the steepness of a ramp?**  Use graph paper, calculators, or online software for graphing.  Ask students to consider the following situation:  *a series of four ramps depicting slope first ramp:  a right triangle with a horizontal length of 3 units and a vertical height of 2 units. second ramp: Two congruent triangles, placed diagonally so that their hypotenuses form a straight line. thrid ramp: Two congruent triangles, placed diagonally so that their hypotenuses form a straight line- two dotted line are added to teh space between the triangles to make a rectangle. fourth ramp:  a triangle comprised of two smaller right triangles and a rectange.*   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |   Ask students to think about how one could find the slope of the line shown. Have students turn and talk with a partner.  Note: There is no need to find slope using a formula. Students can find slope by forming a ratio of the y-values to its corresponding x-value for any point on the line. (ex. 2/1, 4/2, 8/4, 1/.5 etc).  Have students simplify the ratios and note the conventional way to write the slope is simply 2. Then have students write the equation of the line and compare the ways slope is represented in each. In the equation y = mx the slope is m; on the graph it is the ratio of the coordinates of the point (x,y) y to x (rise to run); in the table it is the ratio of the y to x or dependent to independent variable.    **Part B Slope of y = mx + b**  Problem:  *Each week George put 3 dollars away to save up some money to buy a new video game. His friend Steve is doing the same thing, putting 3 dollars away a week. Steve already had 12 dollars when they both started their savings plan.* *The graph depicts each of their savings. Which line represents George? Steve? Explain your reasoning*.  **Two parallel linear functions. One function starts at (0, 4) and has a slope of 1. The second starts at the origin.**    Have students note the steepness of the lines. How does the value of *m* affect the steepness?  Introduce the word slope as the numeric value that is used to describe the rate of change for a linear function.  How can they change the steepness?    The change in steepness as in slope can be done either by vertical changes or by horizontal changes or by both.  Maintaining steepness in slope requires maintaining proportions.  Each step horizontally is 1 week. Each step vertically is $3.00. Students should note that the Steve’s line is a vertical translation of Greg’s line by 12 units. The relationship represented by George’s savings is proportional, but the relationship represented by Steve’s is not.  Ask students to list several ordered pairs (x,y) for George’s line and determine a relationship between the ordered pairs. (ex. (3,9),(6,18) etc. are all related by a factor of 2). Both x and y coordinates of the point (3, 9) on George’s line are twice the corresponding coordinates for (6,18). Also on George’s line, every point represents the how much he saves each week by forming a ratio of y to x, y/x=3/1=6/2=21/7 etc.  Is there a similar relationship in Steve’s line between ordered pairs? Can we determine the equation of the line that represent’s Steve’s savings?  **Activity 2 Determining slope for lines that are not proportional relationships**  Remember that the slope of Steve’s line is a measure of his change in savings. His change in savings is a ratio of amount saved to number of weeks he saved.  Ask students: *What does the point (4, 24) represent on Steve’s graph? What does the point (7, 33) represent?*  We know Steve is also saving at a rate of $3/week. The point indicates that he had 24 dollars in savings in week 4. He had 33 dollars in savings in week 7.  The elapsed time is 7 – 4, or 3, weeks. During these three weeks he saved 33-24, or 9, dollars. So his rate of savings is the ratio of 9 dollars to 3 weeks, or 3 dollars per week.  Try this with 2 different points on Steve’s line.  There is no direct variation in Steve’s weeks and savings.  The equation for George’s line is y = 3x. *What is the equation for Steve’s line?*  Emphasize *the rate of change is the slope of the line*.  Note: Every point on Steve’s line is a vertical translation of George’s line by 12 units  Y = 3x + 12.  **Closure**  Exit Ticket- Explain how to find slope from two known points.  **Extended Learning/Practice (homework)**  Teacher should provide practice problems for finding slope based on two known points and using similar triangles.  **Teacher Reflection (to be completed after lesson)**  What went well in this lesson? Did all students accomplish the outcome(s)-*Using Similar Triangles to Determine Slope?* What evidence do I have? What would I do differently next time? A distance/time graph (distance is the vertical axis and time is the horizontal axis)  of a line depicting the walking rate of Bob.  The  line that passes through the origin and the point (3,10). Formative Assessment (1 day)Assessing Student Understanding for the Different Representations of Linear Relationships **Guiding Question:** *Can students interpret unit rates given in different units to determine a better buy?*  **Standard(s)/Unit Goal(s) to be addressed in this assessment:**  **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.  ***SMP1*** - Make sense of problems and persevere in solving them  **SMP2** Reason abstractly and quantitatively  **SMP3** – Construct viable arguments and critique the reasoning of others  ***SMP4*** - Model with mathematics  **SMP6**- Attend to precision  **By the end of this assessment students will know and be able to:**  Understand that proportional relationships share the same visual characteristics  Compare and graph different unit rates.  **Anticipated Student Preconceptions/Misconceptions**  Students will not notice that there’s a difference in the rate – one plan is per month and the other is per year. This will be OK for the first two graphs in the formative assessment but will become tricky when putting both on the third graph.  **Assessment Description**  Introduce the following task by giving the students the following prompt (see attached worksheet):  Your family wants to subscribe to an Internet Instant Movie plan. They researched Mulu and Tamacon plans. They found Mulu charges $7 per month after an initial subscription fee of $20. Tamacon charges $90 per year.   * Graph each equation on separate coordinate planes (see worksheet). * Write an equation for each plan. * Which plan is more cost effective for year 1? Explain your reasoning. * Which plan is more cost effective after year 2? Explain your reasoning. * Which plan should your family subscribe to? Explain your reasoning. * On a third coordinate plane, making sure that your units align, graph both plans together on one coordinate plan. * Would you change your plan? Write a paragraph to explain your answer. Be sure to use your mathematical evidence from the problem to support your answer.   **Closure**  Review outcomes of this lesson:   * Demonstrate an understanding of unit rate and slope for the Mulu and Tamacon Internet movie plans. * Write an equation for both the Mulu and Tamacon Internet movie plans.   Make an argument for choosing one plan over another by using mathematical evidence to support their reasoning. |

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| **Lesson 5 Formative Assessment**  **Your family wants to subscribe to an Internet Instant Movie plan. They researched Mulu and Tamacon plans. They found Mulu charges $7 per month after an initial subscription fee of $20. Tamacon charges $90 per year. Which plan should they choose? They hope to keep the plan for several years.** | | |
| Use graphs, tables, and equations to help you make your decision. | | |
| Answer the reflective questions below to assist you in your decision making:  Which plan is more cost effective for year 1? Explain your reasoning.    Which plan is more cost effective after year 2? Explain your reasoning.  Which plan should your family subscribe to? Explain your reasoning.  E. On a third coordinate plane, making sure that your units align, graph both plans together on one coordinate plan.  F. Would you change your plan? Write a paragraph to explain your answer. Be sure to use your mathematical evidence from the problem to support your answer. | | |
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Content Area: **Mathematics** Grade(s): **8** Time **60mins**

**Unit Title:** Proportions, Lines, and Equations

# Lesson #6 Title: Deriving the equations y= mx and y=mx+ b

**Essential Question(s) to be addressed in this lesson*:***

*How can similar right triangles help us understand the slope of a line?*

***Guiding Question to be addressed in this lesson:***

*Why do linear functions have equations that look like y=mx and y=mx + b*

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at b.

**8.F.3** Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

**SMP2** Reason abstractly and quantitatively

***SMP4*** Model with mathematics

**Outcome(s)**

**By the end of this lesson students will know and be able to:**

*Determine slope of any non-vertical line through the use of similar triangles.*

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| **Lesson Sequence and Description** |
| Recall Steven’s and George’s savings plans and their equations from lesson 4 to activate the lesson.  Make a table of the x and y values for a line AB for 5 points.   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | x | 0 | 1 | 2 | 3 | 4 |  |  |  |  | | y | 0 | 12 |  |  |  |  |  |  |  |   What would the value of y be if x = 10? (add it to the table)  What would the value of y be if x = r? (add it to the table)  What would the value of y be if x = x? (add it to the table)  How can you use the last answer to write a rule for finding y from x?  What meaning does 12 have in this equation?  Make a table of the x and y values for another line GH for 5 points.   |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | x | 0 | 1 | 2 | 3 | 4 |  |  |  |  |  | | y | 5 | 17 |  |  |  |  |  |  |  |  |   What would the value of y be if x = 10? (add it to the table)  What would the value of y be if x = r? (add it to the table)  What would the value of y be if x = x? (add it to the table)  How can you use the last answer to write a rule for finding y from x?  What does the 12 represent in the equation in terms of the line? What meaning does the 5 have?  What else do you notice when comparing the equations for lines AB and GH?  -----------------------------------------  What do you think would be the equation for a new line, LM that is a 10-unit translation above line AB? How about 10 units below?  Complete the table of the x and y values for line.   |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | x | 0 | 1 | 2 | 3 |  | 5 |  |  |  |  |  | | y |  | 8 |  |  |  | 0 |  |  |  |  |  |   What would the value of y be if x = 0? (add it to the table)  What would the value of y be if x = 10? (add it to the table)  What would the value of y be if x = x? (add it to the table)  How can you use the last answer to write a rule for finding y from x?  Can you see the value of slope in this equation? What is different about the slope for this line versus the lines AB and GH?  **Extended Learning/Practice (homework)**  Homework: MARS Lesson S1 – found at: http://map.mathshell.org/materials/lessons.php?taskid=440&subpage=concept  **Teacher Reflection (to be completed after lesson)**  What went well in this lesson?  Did all students accomplish the outcome(s))?  *Using Similar Triangles to Determine Slope*  What evidence do I have? What methods did the students convey in their exit tickets?  What would I do differently next time? |

The graph compares distance from starting place in yard(s) versus time in seconds (t). Emma starts at the origin and intersects Maggie at (10, 50). Her graph leaves the page at (14, 70). Maggie starts at (0, 30) and the graph leaves the page at (20, 70). 


MARS Mathematics Assessment Project <http://map.mathshell.org/materials/lessons.php?taskid=440&subpage=concept>

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**Content Area/Course:** Mathematics **Grade(s):** 8 **Time:** 60 mins

**Unit Title:** Proportions, Lines, and Equations

# Lesson #7 Title: Determining an equation of a line given 2 known points

**Essential Question(s) to be addressed in this lesson*:***

How can similar right triangles help us understand the slope of a line?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at b.

**8.F.3** Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

**SMP2** Reason abstractly and quantitatively

***SMP4***  Model with mathematics

**SMP 6** Attend to precision

**Outcome(s)**

***By the end of this lesson students will know and be able to:***

*Write the equation of a line in the form y= mx + b from two known points.*

**Instructional Resources/Tools**

**MARS Lessons:** [**www.scoe.org/files/mars-grade8.pdf**](http://www.scoe.org/files/mars-grade8.pdf)

**Anticipated Student Preconceptions/Misconceptions**

The line has to go through the origin to make an equation. We have to know the y-intercept to make an equation. We need a table of values to determine slope.

**Formative Assessment**

The homework from Lesson 1 Handout 5.2.1 . Be sure to review it with students so you can begin the derivation of y=mx+b.

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| **Lesson Sequence and Description**  Teacher Note: This lesson touches upon the 8.F.4 standard in the Functions domain of the Common Core Mathematics Framework:  1. Review the previous night’s homework by having students share their methods for determining the equation for Maggie’s line. The students should be able to provide the equation y=2x+30.  2. Be sure the graph is displayed on the Smart Board or Whiteboard with points (0, 30) and (10, 50) labeled and the equation y=2x+30. Begin the process of deriving y=mx+b through explicit instruction.   * Ask students to identify the output (y) for x=3. (answer: 36) Label this point on the graph. * Ask students to calculate the slope using the points (3,36) and (10, 50). Confirm that the slope is 2. [answer: (50-36)/(10-3) = 2/1] * Now tell students that we will pretend that we don’t know the y-intercept. * Tell students that there must be another point on the line (x,y) that we can use to help determine the slope. Ask students to calculate the slope using the points (x,y) and (10, 50). [answer: (50-y)/(10-x) = 2/1] * Ask students if they recognize the above as anything that they’ve seen before? [answer: proportion] * Question students through the solution of the proportion (50-y)/(10-x) = 2/1. * Connect the resulting solution y=2x+30 to their solutions from the homework. * Have students work in pairs to use this new method to find the equation of the line that passes through (5,8) (7,14) and (x,y). Have them first calculate slope from the two known points and setting that value equal to the slope calculation with (x,y) and one of the known points. Solve that proportion and put it in the form y=mx+b. * Ask 2 pairs of students to compare their solutions (work in groups of 4). * Call the class back to share their discussions of the solution. Did the pairs have the same solution? How did they determine the correct solution when they had two different answers? * Set the students back into their original pairs to work on three additional problems.   1. (9,4) and (6,2) [answer: y=2/3x-2]  2. (3,8) and (6,-4) [answer: y= -4x+20]  3. (-2,2) and (5,-4) [answer: y= -6/7x +2/7]  NOTE: This may need scaffolding support on this problem! |
| Closure  To end the lesson, have students share their answers to the last three problems of the lesson. You may need to provide scaffolding support or explicit instruction.  1. (9,4) and (6,2) [answer: y=2/3x-2]  2. (3,8) and (6,-4) [answer: y= -4x+20  3. (-2,2) and (5,-4) [answer: y= -6/7x +2/7]  Exit Ticket: What do you think are the most difficult steps of writing an equation of a line from two known points?  What do you think are the easiest steps of writing an equation of a line from two known points? |

# Lesson 8 Title: *EXTRA PRACTICE*

**Time: 60 minutes**

**Essential Question(s) to be addressed in this lesson*:***

*How can similar right triangles help us understand the slope of a line?*

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at*b*.

**8.F.3** Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

*SMP1* - Make sense of problems and persevere in solving them

SMP2 Reason abstractly and quantitatively

*SMP4* - Model with mathematics

SMP 6- Attend to precision

**Outcome(s)**

***By the end of this lesson students will know and be able to:***

* Use rise-run triangles to determine the slope of a line
* Determine if the slope of a line is positive or negative
* Express the slope of a line as a fraction

**Instructional Resources/Tools**

Rise-Run Triangles: <http://illuminations.nctm.org/LessonDetail.aspx?id=L728>

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| **Lesson Sequence and Description** Illuminations: Rise-Run Triangles <http://illuminations.nctm.org/LessonDetail.aspx?id=L728>  To start the lesson, ask students what they already know about slope. They may know terms such as *rate of change* and *rise over run*. Often, students have recollection of these terms but don't remember or understand what they mean or how they relate to *slope*.  Ask students what it means to have *positive* or *negative* slope. Encourage a student to come to the front of the room and draw a line with positive slope. Ask classmates if they agree that the line has positive slope, and then ask how they can tell.  A line with positive slope is pointing upward as you look to the right. You always want to see if the line is pointing upward or downward on the *right* side of the graph, just as we read to the right.  Sketch these two lines with positive slope for students to see.  This image depicts two graphs. The first graph shows a linear function with a positive slope. It moves from the third quadrant through the second and to the first, without passing through the origin. The second graph has a positive slope, but it  is not as steep.  Ask students to tell you all they can about the two graphs. What's the same? What's different? Emphasize that although both lines have positive slope, there is something different about the direction in which they point. Explain that this description of how *slanted* a line is can be described by a number called its *slope*.  Now, draw a third line that has the same slope as the first line, but a different *y*-intercept. Ask students again for comparisons.  This image shows three graphs. The first and second match the previous image  but the third passes from the third quadrant, through the fourth, and into the first.  Students should eventually recognize that the third line has the same slope as the first line. Once they do, they are ready to think about the **slope number** as a description of how slanted a line is.  Use the [activity sheet](http://illuminations.nctm.org/lessons/6-8/SlopeTriangles/JustSlope.pdf" \t "_blank) for practice and enforcement.   |  |  | | --- | --- | | Image of the activity sheet referenced in the link to the image's left. | [Counting for Slope](http://illuminations.nctm.org/lessons/6-8/SlopeTriangles/JustSlope.pdf" \t "_blank) |   The activity sheet guides students through a process for finding the slope of a given line. Page 1 is meant to be completed as a class, so having an overhead slide of this page will be helpful.  Distribute the activity sheets and make sure each student has 1 or 2 colored pencils. Many students enjoy using a colored pencil to draw and shade the slope triangle, and doing so makes the lesson more memorable. You might ask students to use one color when they're drawing the triangle for a line with positive slope, and another color for triangles representing negative slope.  Shade in the slope triangles with students as shown below.   |  |  |  | | --- | --- | --- | | Graph displaying linear function  passing through (0, 1) and (-2, 0). There are two similar triangles whose hypotenuses correspond with the function. The first has height 2 and base 3 and the second has height 4 and base 6. | This graph shows a linear function passing through points (0, -3), (1, 0). There are two similar triangles again; the first has base 1 and height 3. The second has base 3 and height 9. | This graph shows a decreasing linear equation passing through the point (0, -3). There are two triangles, one overlayed over the other. The smaller triangle has base 2 and height 4, the second has height 8 and base 4. |   Encourage students to simplify their fractions on page 1 of the activity sheet. Point out that for each line, the simplified forms of the fractions are equivalent — no matter which two points on the line you student use, or how large the triangle is, you get the correct slope.  On page 2, students are given the slope triangle in the first 3 examples (the top row). In the next 3 examples (middle row), they are given only the points to use to draw the triangle. In the last 3 examples (bottom row), students have to find the points themselves before drawing the triangle and determining the slope. The idea here is to gradually get students comfortable with finding the slope.  While students work on page 2, be sure that they:   * Simplify all fractions * Determine which lines have negative slope and use a negative fraction to represent the slope of these lines.   This exercise provides students with the skill of finding the slope of a line from a graph. This skill can be applied to less abstract examples using real data from a table or a graph. |
| Extended Learning/Practice (homework)  **4. SLOPE** Consider the line shown in the graph.  **a.** Draw two triangles that show the rise and the run of the line using points *A* and *B* and points *M* and *N*.  **b.** Use the triangles to find the slope of the line.  **c.** Repeat parts (a) and (b) using different pairs of points.  **5. REASONING** You draw a triangle that shows the slope of a line using two points. Then you draw another triangle that shows the slope using a different pair of points on the same line. Are the triangles similar? Explain.  **6. WRITING** Explain why you can find the slope of a line using any two points on the line.  *This graph shows a decreasing linear function. There are four points marked: A (1, 11), M (3, 7), N (4, 5) and B (6, 1).*  **Teacher Reflection (to be completed after lesson)**  What went well in this lesson?  Did all students accomplish the outcome(s)?   * Use rise-run triangles to determine the slope of a line * Determine if the slope of a line is positive or negative   What evidence do I have?  What would I do differently next time? |

**Content Area/Course:** Mathematics **Grade(s):** 8 **Time:** 2-3 days

**Unit Title:** Proportions, Lines, and Equations

**Lesson 9 Title:** **Summer Work - Comparing Jobs**

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| ***CURRICULUM EMBEDDED PERFORMANCE ASSESSMENT***  **Title: Summer Work - Comparing Jobs**  **Goal**: Your job is to research different summer work opportunities and determine which opportunities represent the most feasible jobs for you to meet your personal goal.  **Role:** Analysts  **Audience:** Your parents  **Situation:**   1. You have asked your parents if you can to go to the NASA Space Camp in Huntsville Arizona next year. Because the camp is expensive (initial costs for Space Camp start at $779 plus transportation to and from your home), your parents said you would need to earn and save the money pay for it. Since you don’t have much money, you want to get a summer job. In order to help you reach your goal, your parents have offered to match the amount of money you earn over the course of the summer. (This means they will give you the same amount of money you earned at the end of the summer). 2. In order to know how much money you need to save, you need to estimate your costs for Space Camp. You need to consider travel to and from home by plane, and taxi for ground transportation to and from the airport. Are meals included in the cost of the camp? If not, you will need to consider money for meals. 3. In order to find the best job, you are to select *at least* 3 jobs from a list of summer job opportunities in the *Summer Opportunities Flyer.* Your task is to analyze these job opportunities using mathematical reasoning of linear relationships. You must include multiple representations (equations, tables, and graphs) for each job situation. 4. You will need to decide which job will allow you to earn the most money and support your decision with evidence of mathematical reasoning of linear relationships.   **Product/Performance:** Construct an analysis of the three job opportunities containing the following information:  A) Show and explain the income potential of at least three different job opportunities.  B) Show and explain which of the three (or more) job opportunities will pay the most over the summer. Be sure to justify your reasoning!  C) Which job will be the best job for you? Explain why this is the case using the mathematical evidence youcollected in this performance task as well as your own interests and goals.  D) Present the information to your parents using varied media to convince them the job you’ve chosen is the best one for you. |

**Summer Job Opportunities Flyer**

**The Brownsville Youth Council has compiled this list of local job opportunities for students.**

1. Yard help needed! I need someone to mow my lawn and my neighbor’s lawn every Friday from June through the end of August. I will pay $8.00 per hour but you need to provide your own lawnmower and gas. It typically takes about 3 1/4 hours to mow my lawn and 2 ½ hours to mow my neighbor’s lawn.
2. Help needed picking vegetables on my farm from 6:00 AM to 10:00 AM Wednesday and Sunday to prepare for the local Farmer’s Market. Pay will be $6.50 per hour and you can take a bag of vegetables home each day.
3. Babysitter needed for my daughter from 8:00 AM to 1:00 PM each weekday. I will pay $6.25 per hour.
4. Paper route available- 75 Monday through Saturday customers. Of the 75 Monday through Saturday customers, 50 are also Sunday customers. The paper carrier earns $0.23 per paper each day for Monday through Saturday delivery and $0.50 per paper for Sunday delivery.
5. Mowing help needed. I need my lawn mowed every week from June through August. I will pay $35 for each mowing. You can use my mower and gas, but if you break it you must pay the repair cost!
6. I need a babysitter for my three young children from 7:00 AM to 12:00 PM each weekday. I will pay $32 per day plus snacks.
7. Energetic dog needs a walker! Will pay $11 per walk. We would like our Chihuahua walked twice a day, once in the morning and once in the evening, on Mondays, Tuesdays, Thursdays and Fridays.
8. Wanted- Car and Bicycle Washer! Our family of six people drive in three vehicles (a car, SUV and jeep) on dirt roads every weekend and would like them washed Monday mornings throughout the summer. Will pay $22 per vehicle to be washed and an extra $5 for each bicycle too, but you need to supply your own soap and rags.
9. Summer help needed every weekday throughout the summer for light housework (vacuuming, dusting, cleaning the bathroom, etc.) and outdoor work (weeding, mowing the lawn, and raking). After the first 5 days you will earn $110. After that, you will earn $200 every 10 days.

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| **Summer Earnings Performance Task Rubric**  **Connecting Proportions, Lines and Linear Equations**  Student Name:     Teacher Name: | | | | |
| ***CATEGORY*** | 4 | 3 | 2 | 1 |
| ***Analysis*** | More than 3 jobs are accurately analyzed with full consideration of all scenario parameters. | 3 jobs are accurately analyzed with evidence of consideration of all scenario parameters. | Less than 3 jobs reanalyzed and/or inaccuracies are evident in mathematical reasoning. | Less than 3 jobs are analyzed with evidence of numerous inaccuracies in mathematical reasoning. |
| ***Mathematical Concepts*** | Work shows evidence of in-depth understanding of proportional relationships, rate, and slope of lines and the relationships between them. | Work shows evidence of full understanding of proportional relationships, rate, and slope of lines and the relationships between them. | Work shows evidence of partial understanding of the concepts- proportional relationships, rate, and slope of lines and the relationships between them. | Work shows evidence of limited understanding of proportional relationships, rate, and slope of lines and the relationships between them. |
| ***Representation*** | Multiple representations (graphs, equations, and tables) is used for all jobs analyzed. | Multiple representations (graphs, equations, and tables) is used for most of the jobs analyzed. | Multiple representations (graphs, equations, and tables) is used for some of the jobs analyzed and/or used only two representations for the jobs analyzed. | Only one representation is used for analyzing the jobs or does not use graphs, equations, and tables to analyze jobs. |
| ***Precision of Mathematical Language*** | Complex mathematical language is accurately used (proportional, linear, slope, etc.) throughout the presentation to communicate about mathematical reasoning and justification of chosen job. | Appropriate mathematical language is accurately used (proportional, linear, slope, etc.) in much of the presentation to communicate mathematical reasoning and justification of chosen job | Some mathematical language (proportional, linear, slope, etc.) accurately used to communicate mathematical reasoning and justification of chosen job. | Limited or no mathematical language is accurately used to communicate mathematical reasoning and justification of chosen job. |
| ***Mathematical Accuracy*** | All work is shown and contains little or no errors in calculations, equations, graphs, and tables. | All work is shown and mostly accurate, but may contain several mathematical errors in calculations, equations, graphs, and tables. | Most work is shown and contains a number of inaccuracies in calculations, equations, graphs, and tables. | Work is limited, missing and/or contains numerous mathematical errors in  calculations, equations, graphs, and tables. |
| ***Argument*** | Strong reasons for selecting the job are included in the presentation and are convincing, thoughtful, and backed up with extensive evidence. | Reasons for selecting the job are included in the presentation, and are convincing and backed up with appropriate evidence. | Some reasons for selecting the job are included in the presentation. Reasons are somewhat convincing and are backed up with some evidence. | Reasons for selecting the job may or may not be included in the presentation. The reasons are unconvincing and/or are backed up with little evidence. |

**Additional Lesson Resources**

**Content Area/Course:** Mathematics **Grade(s):** 8 **Time:** 60 mins.

**Unit Title:** Proportions, Lines, and Equations

**Optional Lesson Title:** Proportional versus Non-proportional Figures

**Essential Question(s) to be addressed in this lesson:**

*What does it mean to be proportional?*

*What does it mean to not be in proportion?*

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

**Assumptions about what students know and are able to do coming into this lesson (including language needs):**

This lesson activates the students’ prior knowledge about making decisions about the proportionality of drawings. It will review scale factor and its role in proving proportionality.

*Teacher Note:* Distinguish between student understanding of the concept and knowledge of the vocabulary words by providing options for students to express their understanding. Students who do not understand the concept will need different support than those who understand the concept but do not have adequate mathematical language to express this understanding either verbally or in writing.

**Outcome(s)**

*By the end of this lesson students will know and be able to:  
Explain the difference between proportional or non-proportional figures.*

**Instructional Resources/Tools**

Software download: <http://www.geogebra.org/cms/download> or

GeoGebra Interactive Dilations

Lesson2.part1

Lesson2.part2

Lesson2.part3

Lesson2.part4

Index cards

**Anticipated Student Preconceptions/Misconceptions**

Students may believe if you have a **relationship (between two quantities)** then it must be a proportional relationship

**Assessment** - Pre-assessment/ Formative

*Think-Pair-Share Activity (see below)*

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| **Lesson Sequence and Description**  Launch the lesson with the essential question: ***What does it mean for quantities to be in proportion? What does it mean to be non- proportional?***  Note: Be careful of the term **non**-proportional. Misconceptions can happen about non-linear proportions (Direct proportions) and non-proportional relationships. Proportional relations can vary directly (linear) (y/x = k, y=kx ) or **non**-directly (inversely) xy = k, y=x/k). Though students in grade 8 do not need to study inverse proportions they should know that other proportional relationships exist (that are non-linear) and will be studied in future courses.  I. Conduct a *Think* (individually for 2 minutes), *Pair* (have students turn and talk to a partner for 2 minutes to brainstorm examples around the essential question), and *Share* (pairs share out their ideas in fours). Hand out index cards to gather results on individual student’s understanding. Have students divide the card in half. During “think” time students will individually write their thoughts about the question at hand on the left side of the index card. They can then use their generated ideas during “pair” time.  Sharing of ideas is done as a whole class to generate examples that are proportional and examples that are not proportional. Collect these ideas on chart paper, Smart board or on your whiteboard.  *Note: Near the end of the class, have students write their thoughts about how/if their thinking has changed as a result of class discussion on the right side of the card. Teacher collects the index cards as an “exit ticket” as students leave the room*  II. **Scaling Up/Down Activity-** compare proportional/non-proportional figures.  Introduce the vocabulary term: dilate - to make or grow larger or wider  Resources: GeoGebra Interactive Dilations  Lesson2.part1  Lesson2.part2  Lesson2.part3  Lesson2.part4  Download Geogebra software: <http://www.geogebra.org/cms/download> You may either download the software to your computer or work with an applet without installing the software.  **Purpose:** To activate prior knowledge of scale factor and proportionality between similar figures. (This will help develop the slope triangle concept in 8.EE.6)  ***If you cannot access the GeoGebra software, please refer to the description of the lesson below:***  **GeoGebra**  **Lesson 2. Part1** starts with a quadrilateral drawn with the following vertices: A(0,0) B(-1,2) C(1,3) D(2,2).   1. Transform/Enlarge this figure by a scale factor of 3 on the graph (click on “Tools” button, then “Transformations Tools”, “Dilate Object from Point by Factor”, then type in “3”).   B. Transform this figure by a scale factor of 0.5 on the graph.  C. What do you notice about each of the transformations? Explain each image.  **GeoGebra Lesson 2.part2:**  A. Plot these points on the graph to make an original figure:  A(0,0) B(-1,3) C(4,2) D(5,1).  B. Plot the following points to make its image:  A1(0,0) B1(-1.5, 4.5) C1(6,3) D1(7.5,1.5).  C. What is the scale factor used to transform the original figure to the image figure?  **GeoGebra Lesson 2.part3:**  A. Plot the following points to form an original figure:  A(0,0), B(-1,3), C(4,4), D(5,1).  B. Plot the following points to make an image of the original:  A1(0,0), B1(-2,3), C1(8,4), D1(10,1)  C. Do you think the image is proportional to the original?  Explain why or why not using mathematical language in your answer.  **GeoGebra Lesson 2.part4:**  A. Plot the following points to form an original figure:  A(0,0) B(-1,3) C(4,4) D(5,1).  B. Plot the following points to make an image of the original:  A1(0,0), B1(-2,6), C1(8,10), D1(10,2).   1. Is this image proportional to the original? Explain why or why not using mathematical language in your answer. |

1. Function notation is not required in grade 8. [↑](#footnote-ref-1)